A time-average model of the RF plasma sheath was developed. The ion "fluid" equations were used with a frictional force to account for ion-neutral collisions. Consideration of the collision dynamics showed that the frictional force may be taken as proportional to the square of the ion drift velocity. The sheath model was used to investigate the ion energy and flux on the electrodes of plasma reactors. The dimensionless quantity Co (collision number) was found to be important in describing the ion motion in the sheath. An analytical expression for the ion bombardment energy was derived, in terms of Co and the sheath voltage, for the range of parameter values typical of high pressure (~1 torr) diode plasma etchers. An application of the model to an oxygen discharge in a parallel plate reactor was considered. Over the parameter range investigated, the ion bombardment energy was found to be only a few tens of eV, much lower than typical sheath voltages (~200V). The ion bombardment energy was found to be a function of the sheath electric-field-to-pressure ratio. The model provides a framework that can be incorporated into more general plasma reactor models which consider transport and reaction phenomena along surfaces undergoing etching.

A rigorous approach to solving for the particle transport in the sheath would require consideration of Boltzmann-like kinetic equations for the ions, electrons, and neutrals, coupled to the Poisson equation for the self-consistent electric field. Solution of the coupled integro-differential equations subject to appropriate boundary conditions could yield the corresponding distribution functions. Such an approach is invariably difficult, although progress has recently been made in solving for the spatially dependent energy distribution functions of ions, electrons, and fast neutrals (resulting from charge exchange collisions) in a time-independent sheath (8). Another approach is to employ the Monte Carlo method as demonstrated by Kushner (9). A simplified approach was taken in the present study motivated by the fact that a reasonable sheath model be developed, yet simple enough to be readily incorporated into more general plasma reactor model. Thus, in the present sheath model, the ion "fluid" velocity was found by coupling the equations of continuity and motion of the ion "cloud" to Poisson's equation for the potential distribution in the sheath. Time-averaged quantities were used throughout, i.e. an equivalent dc model was employed. Equivalent dc models of RF sheaths have been used before (10). Their advantage is the simplicity of the calculation. Their disadvantage is that they are not applicable to low frequency (<10 MHz) operation and that they provide only the average ion energy and not the ion energy distribution.

**Model Formulation**

To eliminate edge effects and to reduce the dimensionality of the analysis, the sheath shall be considered as an infinite planar sheet of finite thickness between an infinite planar absorbing surface and a semi-infinite plasma. Within the sheath the medium shall be regarded as two interpenetrating fluids composed of neutral gas molecules and singly charged daughter ions, respectively. In all cases, the neutral fluid will be taken to be in thermal equilibrium at some ambient temperature, T. The ions and the neutrals are of equal mass, m. The electron density will be high enough to be regarded as a smooth potential distribution,

\[ \nabla \cdot \left( n_e u_e \right) = 0 \]  

\[ \nabla \cdot \left( n_i u_i \right) = 0 \]  

\[ \nabla \cdot \left( n_n u_n \right) = 0 \]  

where the vectors \( u_e, u_i, \) and \( u_n \) are the velocities of the electron, ion, and neutral species, respectively, \( n_e, n_i, \) and \( n_n \) are their respective densities.
MODEL OF THE RF PLASMA SHEATH

For mathematical convenience, the differential collision cross section will be assumed to be independent of $g$ (i.e., $\sigma(x, g) = \sigma(x)$) and $\Delta u(x, g)$ will be assumed to be of the form

$$\Delta u(x, g) = - g \cos \theta \xi(x)$$

Physically, this corresponds to a hard sphere description of the ion-molecule interaction. Such a description is plausible since the collisional kinetic energy is generally quite large and the repulsive core of the interaction is dominant.

One can substitute Eq. [5]-[7] into the expression for $F_i$ Eq. [4], and integrate to obtain

$$F_i = -2 \pi m u_i^2 \int_0^\pi d\chi \sin \chi \sigma(\chi) \xi(\chi)$$

Thus, the frictional force is approximately quadratic in $u_i$.

It is convenient to define a frictional coefficient, $\alpha_i$, such that

$$F_i = -\alpha_i u_i^2$$

where

$$\alpha_i = 2 \pi mN \int_0^\pi d\chi \sin \chi \sigma(\chi) \xi(\chi)$$

The coefficient can be evaluated if specific forms are given for $\sigma(\chi)$ and $\xi(\chi)$. For elastic hard sphere interactions

$$\sigma_e(\chi) = \pi D^2$$

$$\xi(\chi) = \frac{1}{2} (1 - \cos \chi)$$

Then, the frictional coefficient takes the form

$$\alpha_i = \frac{\pi mN D^2}{2}$$

Charge exchange collisions can be treated as grazing encounters which, if there were no charge exchange, would have a scattering angle near $\pi$. However, the effect of the charge exchange is to change the scattering angle to near $\pi$ radians (for equal ion and neutral masses). If the constant, $D_{ex}$, is defined as an effective diameter appropriate to charge exchange, the charge exchange cross section can be given an idealized form

$$\sigma_{ex}(\chi) = \pi D_{ex}^2 \delta(\pi - \chi)$$

where $\delta(\pi - \chi)$ is the Dirac delta function. Dynamically, all charge exchange collisions are being treated as if $\chi = \pi$.  

---

\[ \frac{\partial u_i}{\partial t} + u_i \frac{\partial u_i}{\partial x} = \frac{qE}{m} + \frac{F_i}{m} \quad [2] \]

\[ \frac{\partial E}{\partial t} = \frac{q}{\epsilon_0} \quad [3] \]

Here, the coordinate $x$ is in the direction normal to the planar boundary surfaces defined previously. Equation [1] is the ion continuity equation. It contains no source term and, thus, embodies the assumption that the ionization rate is negligible within the sheath in comparison to the plasma or glow region. This assumption is consistent with the assumption of negligible electron density since electron impact is usually the dominant ionization mechanism. Equation [2] is the equation of motion for the ion fluid with $u_i$ denoting the average ion velocity. The two terms on the right-hand side account, respectively, for the effect of the sheath electric field and the effect of the dynamic frictional force, $F_i$, due to interaction with the medium. Equation [3] is Poisson’s equation which relates the divergence of the electric field to the space charge density.

Dynamic friction.—The preceding set of equations is closed in all quantities except the frictional force, $F_i$. In this section, a form for $F_i$ as a function of molecular dynamic parameters and the average ion velocity, $u_i$, will be obtained. To begin, one notes that within a slightly ionized discharge, the great majority of collisions experienced by any given ion are binary encounters with a neutral gas molecule (shown schematically in Fig. 1). Phenomenologically, one can express the dynamic frictional force as an average over all possible collisions of a “test” ion moving at a velocity, $u_i$, with a thermal distribution of neutrals. Restriction of consideration to spherically symmetric ion-molecule interactions yields the expression

$$F_i = -\frac{m}{2} \int_0^\pi d\chi \sin \chi \int dg g \sigma(x, g) \Delta u(x, g) f(v)$$

where $g$ is the relative velocity of the test ion with respect to a gas molecule ($g = |g|$) and $\chi$ is the differential scattering angle (see Fig. 1). The function, $\sigma(x, g)$, is the differential collision cross section for the ion-molecule interaction and $\Delta u(x, g)$ is the collisional change in the component of the ion velocity parallel to the X-axis (i.e., normal to electrode or wall surfaces). The molecular velocity distribution function, $f(v)$, is taken to be a Maxwellian

$$f(v) = N \left[ \frac{m}{2 \pi kT} \right]^{3/2} e^{-mv^2/2kT}$$

where the molecular velocity magnitude, $v$, is

$$v = \sqrt{g^2 - 2gu_i \cos \theta + u_i^2}$$

and the angle $\theta$ is taken between the relative velocity $g$ and the X-axis.
The differential cross section including both elastic and charge exchange collisions is \( \sigma = \sigma_{el} + \sigma_{ex} \). The form of \( \sigma(x) \) remains as in Eq. [13]. When charge exchange is included, \( \alpha \) becomes

\[
\alpha = \frac{Nm\pi}{2} (D^2 + D_e^2) \tag{16}
\]

which suggests that an ad hoc general expression for \( \alpha \) can be given as

\[
\alpha = \frac{Nm}{2} \sigma \tag{17}
\]

Model equations and boundary conditions.—Equations [1]-[3] can be simplified by replacing the instantaneous field with its time average \(<E>\). Taking into account Eq. [10] and that

\[
<E> = -\frac{dV}{dx} \tag{18}
\]

yields

\[
\frac{d}{dx} (n_u u_i) = 0 \tag{19}
\]

\[
u_i \frac{d u_i}{dx} = -\frac{q}{m} \frac{dV}{dx} - \frac{\alpha n_i^2}{m} \tag{20}
\]

\[
\frac{d^2 V}{dx^2} = -\frac{q n_i}{\epsilon_0} \tag{21}
\]

It now remains to impose suitable boundary conditions on this system of equations. Bohm (27) demonstrated that ions entering the sheath must move with an approximate velocity which is determined by the plasma parameters characteristic of the glow. If one defines the coordinate origin, \( x = 0 \), to be at the sheath-glow interface (Fig. 2), the Bohm criterion in a collisionless plasma is

\[
u_i(0) = \sqrt{\frac{kT_e}{m}} \tag{22}
\]

Here, \( T_e \) is the electron temperature within the glow region. Typically, in an RF discharge of practical importance, \( kT_e \) is from 1 to 10 eV.

The essence of the Bohm criterion is that before entering the sheath, an ion falls through a potential drop of order, \( kT_e/2q \), which "leaks" into the glow from the sheath. A potential drop of this magnitude can be sustained within the glow due to the thermal energy of the electron fluid without grossly perturbing the system away from ideality (i.e., the glow remains quasi-neutral with approximately equal ion and electron densities). Bohm’s criterion must be modified when the negative ion density in the glow is large as compared to the electron density.

If one adopts the convention that \( x < 0 \) corresponds to the glow and \( x > 0 \) corresponds to the sheath (and/or electrode), one can modify the Bohm criterion to include the effect of dynamic friction as follows

\[
u_i(0)^2 = \frac{kT_e}{2} - \int_{-\infty}^{\infty} dx |V(x)| \tag{23}
\]

Thus, the kinetic energy of an ion entering the sheath approximately equals the potential energy, \( kT_e/2 \), minus energy lost to friction against the medium. In practice, the quasisteady state cannot be evaluated and will be replaced by an estimate, \( \lambda_0 F_0(0) \), where \( \lambda_0 \) is the Debye length. The Debye length is a characteristic measure of the distance within a quasi-neutral plasma over which a variation in potential of order, \( kT_e/2q \), is effectively screened out. Consequently, electric fields within the plasma should be no greater than the order of \( kT_e/2q \lambda_0 \). Hence, before entering the sheath, an ion is strongly affected by electrical and frictional forces primarily within the last Debye length of the glow (presheath in Fig. 2) before reaching the sheath-glow interface. Since the average electron energy in the glow associated with the \( x \)-degree of freedom is \( kT_e/2q \), the sheath-glow interface is the plane at which the electron average energy is reflected back to the glow, leaving the sheath devoid of electrons, as assumed earlier.

It should be mentioned at this point that the above discussion associated with the Bohm criterion is most appropriate for a dc sheath. For example, periodic electron "leakage" into an RF sheath, to neutralize the positive ion flux to a capacitively coupled electrode, would violate our assumption of the sheath being devoid of electrons. However, it was felt that, for the present simplified sheath model, the Bohm criterion may be used (as was done in previous work on RF sheaths, Ref. (22)) to provide an approximate boundary condition for the ion current at the glow/sheath interface.

In the present work, the bulk ion density \( n_i \) was taken to be constant within the glow up to the glow-sheath interface (\( x = 0 \)). Thus, the Bohm criterion and quasi-neutrality provide the following boundary conditions at the sheath-glow interface

\[
\begin{align*}
\nu_i(0) &= \nu_i^* = \frac{\sqrt{kT_e}}{\sqrt{m}} \\
\nu_u(0) &= \frac{\sqrt{kT_e}}{\sqrt{m + 2n_i}} \\
\frac{dV}{dx}\bigg|_{x=0} &= -\frac{kT_e}{2q\lambda_0} \tag{26}
\end{align*}
\]

\( V(0) \) can be chosen arbitrarily and will be taken to be zero. The electrode is assumed to be an equipotential surface

\[
V(d) = -V_s \tag{27}
\]

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\[
V(d) = -V_s \tag{27}
\]

Equation [27] implies that the electrode has a negative potential with respect to the plasma. The three boundary conditions imposed on potential do not overspecify the problem since the sheath thickness \( d \) is not known a priori.

It is instructive to render the equations and boundary conditions dimensionless. By defining

\[
\begin{align*}
n_i^* &= \frac{n_i}{n_e} \\
\nu_i^* &= \frac{\nu_i}{\sqrt{kT_e}} \\
V^* &= \frac{qV}{kT_e} \tag{28}
\end{align*}
\]
one can rewrite Eq. [19]-[21] to obtain

\[
\frac{d}{dx} (n^* u^*) = 0
\]

[30]

\[
\frac{d}{dx} u^* = -\frac{dV^*}{dx} - Co u^* \frac{V^*}{-n^*}
\]

[31]

\[
\frac{d^2V^*}{dx^2} = -n^* - \frac{dV^*}{dx} \frac{1}{1 + 2Co}
\]

[32]

In rendering Eq. [21] dimensionless, the expression for the Debye length \( \kappa_D \) boundary conditions become

\[
n^*(0) = 1 \quad u^*(0) = \sqrt{\frac{1}{1 + 2Co}}
\]

[33]

\[
\frac{dV^*}{dx^*} \bigg|_{x^*=0} = -\frac{1}{2} V^*(0) \quad V^*(d^*) = -V^*_s
\]

[34]

The dimensionless quantity \( Co \), hereafter referred to as the "collision number," directly affects the energy of ions bombarding the electrode. With use of Eq. [17] and [29], \( Co \) can be expressed as

\[
Co = \frac{1}{2} N_A p n_l
\]

[35]

\( Co \) is a measure of the number of collisions an ion suffers when traversing the sheath. If the value of \( Co \) is small (\( Co \rightarrow 0 \)), such as under low pressure operation, ions free-fall into the sheath. This is the so-called space charge sheath limit. In such a case ions bombard the electrode surface with energy equal to the average sheath voltage. At the other extreme of high \( Co \) values, ions experience many collisions during their transit through the sheath and ions can be regarded in "equilibrium" with the local field. In that case, ion inertia effects can be neglected (\( du/dx \rightarrow 0 \)), and the ion bombarding energy will be substantially smaller than the sheath voltage.

Combining Eq. [30]-[32] yields

\[
\frac{d^2}{dx^2} (u^* \pm^2) + 2Co \frac{d}{dx} (u^* \pm^2) = \frac{2}{u^*} \sqrt{\frac{1}{1 + 2Co}}
\]

[36]

which was solved for \( u^* \pm^2 \) by a finite difference algorithm. Equation [33] provided \( u^* \pm^2(0) \), one of the two initial conditions required. The second initial condition was found by applying Eq. [31] for \( x^* = 0 \), and using Eq. [33] and [34] to obtain

\[
\frac{du^* \pm^2}{dx^*} \bigg|_{x^*=0} = 1/(1 + 2Co)
\]

[37]

After obtaining the drift velocity \( u^* = u^* (x^*) \), Eq. [30] and [32] were solved simultaneously to find the potential distribution in the sheath. Calculations stopped when the potential attained a value of \(-V^*_s \), i.e., when the electrode was reached, at which point the sheath thickness could be obtained from the corresponding \( x^* \) value.

**Results and Discussion**

Two sets of calculations were carried out, the first being of general behavior in dimensionless parameter space, the second being specific to an oxygen plasma in a parallel plate etching reactor.

Figure 3 shows the dimensionless sheath potential as a function of dimensionless distance from the glow/sheath interface (\( x^* = 0 \)) into the sheath, for several values of \( Co \). The absolute value of the sheath potential increases monotonically with distance into the sheath. For a given distance into the sheath, the potential "drop" is higher for higher values of the collision number. This is because the sheath "resistance" increases with \( Co \). In practice, if one has an estimate of the sheath voltage and of \( Co \), one can obtain \( x^* \) from Fig. 3 and in turn the sheath thickness. For example, in a symmetric discharge (electrode area ratio \( \approx 1 \)), the sheath voltage may be taken as approximately \( 1/4 \) of the applied RF peak-to-peak voltage (assuming that there is negligible voltage drop across the bulk plasma). In a strongly asymmetric system (electrode area ratio \( << 1 \)), the sheath voltage over the smaller area electrode (usually the powered electrode) may be taken as \( 1/2 \) of the applied RF peak-to-peak voltage (11). The collision number is given by Eq. [35] and requires knowledge of the collision cross section, values of which may be found in Ref. (14) and (16).

The value of \( x^* \) obtained in the foregoing manner can then be used with Fig. 4 to obtain the ion velocity (and energy) for the given \( Co \). As seen in Fig. 4, the ion velocity decreases with increasing \( Co \) because ions experience more collisions in the sheath. The ion velocity changes rapidly close to the glow/sheath interface. The current continuity equation then requires that the ion density falls rapidly there. For values of \( Co > 1 \), the ion velocity quickly attains a "fully developed" value, changing only slowly with distance down the sheath. This is the force balance condition for which the electrostatic force and the frictional force almost balance each other, i.e., ion inertia (acceleration) is negligible (\( du/dx \rightarrow 0 \) in Eq. [20]). Then, Eq. [20] yields

\[
u_i = \frac{qe}{\alpha_i}
\]

[38]

or, since \( \alpha_i \) is proportional to pressure \( p \) (Eq. [17])

\[
u_i \sim \frac{E^{1/2}}{p}
\]

[39]

This result was obtained by Wannier (12) for the motion of gaseous ions with fully developed velocity distribution under a strong electric field (\( E/p \gg 1 \) V/torr-cm), and constant mean free path ion-neutral collisions (such as in the case of hard-sphere interactions).

There are some limiting forms of the sheath model which are amenable to analytical solution. In the "frictionless" case (\( Co \rightarrow 0 \), Eq. [30] can be solved analytically to
obtain an explicit expression of ion velocity vs. distance into the sheath

\[ u_i^* = \left[ 4y + \frac{585}{512} + z \right]^{1/3} + \left[ 4y + \frac{585}{512} - z \right]^{1/3} - \frac{7}{8} \]  

\[ \frac{1}{8} \]  

with

\[ y = \frac{9x^{*2}}{16} + \frac{33x^*}{64} \]  

\[ z = 3\left( x^* + \frac{11}{24} \right) \sqrt{y + \frac{29}{64}} \]  

For large \( x^* \), the terms containing \( x^{*2} \) dominate and Eq. [40] is simplified to

\[ u_i^* = \left( \frac{9}{2} \right)^{1/3} (x^*)^{2/3} \]  

\[ \frac{1}{2} \]  

Furthermore, in the frictionless case \( u_i^* = \sqrt{1 - V^*} \). For large \( x^* \), for which [43] is valid, \( -V^* \gg 1 \) (recall that \( V^* < 0 \)), and

\[ u_i^* = \sqrt{-V^*} \]  

Noting that \( I_i = qn_iu_i \), taking into account Eq. [44], and introducing dimensional variables into Eq. [43], results in the high vacuum version of the Child-Langmuir law

\[ I_i = \frac{4e_0}{9} \frac{2q}{m} V^{6/3} \frac{\sigma_i}{d^2} \]  

\[ \frac{1}{9} \]  

Another limiting case results when electrostatic and frictional forces balance each other, i.e., when ion inertia can be neglected. For any finite value of \( Co \), this will happen at sufficiently large \( x^* \). The higher the value of \( Co \), the sooner the ion will attain the force balance condition.

When ion inertia is neglected, the sheath equations can be integrated directly to yield

\[ u_i^* = \left( \frac{3}{2Co \sqrt{1 + 2Co}} \right)^{1/3} (x^* + C)^{1/3} \]  

\[ \frac{3}{2Co \sqrt{1 + 2Co}} \]  

The integration constant \( C \) can be eliminated by using boundary condition Eq. [33]. Then

\[ C = \left( \frac{2Co}{3(1 + 2Co)} \right)^{1/3} \]  

\[ \frac{2Co}{3(1 + 2Co)} \]  

However for the large \( x^* \) for which Eq. [46] is valid, \( C \) can be neglected compared to \( x^* \), and Eq. [46] reduces to

\[ u_i^* = \left( \frac{3}{2Co \sqrt{1 + 2Co}} \right)^{1/3} (x^*)^{1/3} \]  

\[ \frac{3}{2Co \sqrt{1 + 2Co}} \]  

In addition, integration of Poisson's equation yields for the case at hand (and for high \( x^* \))

\[ V^* = \frac{9}{10} \left( \frac{2Co}{3(1 + 2Co)} \right)^{1/3} (x^* + C)^{2/3} \]  

\[ \frac{9}{10} \left( \frac{2Co}{3(1 + 2Co)} \right)^{1/3} \]  

Equation [48] and [49] indicate that for any finite value of \( Co \), \( u_i \) will become asymptotic to a one-third power law and \( V \) will become asymptotic to a five-thirds power law with respect to \( x^* \), as \( x^* \to \infty \). The above limiting cases are displayed in Fig. 5 where the complete solution (solid lines) is compared to the asymptotic solutions. If \( Co = 0 \), the complete solution almost coincides with the Child-Langmuir law, Eq. [43] (dash-dotted line). For \( Co = 1 \), the complete solution is virtually identical to the approximate solution Eq. [46]. In fact, for \( x^* \) greater than about 10, Eq. [48] is a good approximation for values of \( Co \gg 0.1 \).

Consider now the range of values of \( Co \) that would be encountered in practice. For \( O_2^+ \) moving in its parent gas, for example, the total collision cross section has been reported as \( \sigma_i = 5.5 \times 10^{-15} \text{ cm}^2 \) (16). For \( Cl_2^+ \) in \( Cl_2 \), \( \sigma_i = 2.67 \times 10^{-14} \text{ cm}^2 \) (14). Therefore a typical value of \( \sigma_i \) may be taken as \( 5 \times 10^{-14} \text{ cm}^2 \). For a high pressure plasma etching reactor for which \( p = 0.5 \text{ torr} \) and \( T = 350 \text{ K} \), the gas density is \( N = 1.38 \times 10^{16} \text{ cm}^{-3} \); a typical value of \( k_B \) is \( 100 \mu \text{m} \) or \( 10^{-2} \text{ cm} \). Then Eq. [35] yields \( Co = 0.5 \). Further, a typical sheath thickness is a few mm, i.e., \( x^* \) is a few decades. As shown above, under these conditions, Eq. [48] and [49] are good approximation. When \( Co \) is small enough, the complete solution almost coincides with the Child-Langmuir law, Eq. [43]. For \( Co \gg 0.1 \), the complete solution is virtually identical to the approximate solution Eq. [48]. In fact, for \( x^* \) greater than about 10, Eq. [48] is a good approximation for values of \( Co \gg 0.1 \).
approaches. Then, by eliminating \((x^* + C)\) between these two equations, one has an expression for the ion bombardment energy as a function of the sheath voltage, applicable to relatively high pressure high current etching reactors.

\[
(u_*)^2 = \frac{(-10V*)^2}{(2Co)^2(1 + 2Co)}
\]  

Equation [50] provides the dimensionless ion energy as a function of the dimensionless potential down the sheath \((V_s)\), for relatively high values of \(Co\). The ion bombardment energy can be obtained when the sheath voltage \((V_s^*)\) is used for \(V_s\). As an example, consider a sheath voltage \(V_s = -250\) V and a plasma electron temperature \(T_e = 4\) eV. Then \(V_s^* = -250/9 = -26.25\). Assuming \(Co = 0.5\), as calculated above, one finds \((u_*)^2 = 11.4\) and the ion kinetic energy \(\varepsilon_i = 1/2 u_*^2 = 11.4 \times 2 = 22.8\) eV. This would be the ion bombarding energy in order of magnitude. The total ion bombardment energy would be \(\varepsilon_i = 22.8/0.559 = 40.8\) eV (see Eq. [52] below).

Besides considering the general characteristics of the sheath, the model was also applied to an actual plasma etching reactor. The experimental system has been described elsewhere (13). Briefly, an oxygen discharge was used to etch polymeric films in a showerhead parallel plate single-wafer etcher. The substrate wafer was resting on the grounded electrode. The average plasma (and ground sheath) potential \((V_p)\) was estimated from measurements of the RF peak-to-peak applied voltage \((V_{pp})\) and of the dc self-bias \((V_{ds})\) with the equation \(V_p = (V_{pp}/2 + V_{dc}/2)\) (11). The dc bias was small in the approximately symmetric system used. The above equation for \(V_p\) is not applicable when a substantial voltage drop occurs across the bulk plasma, such as under conditions resulting in low electron mobility and/or density. However under typical conditions (-1 torr, 100W) it was found that the plasma resistance (-100) was small compared to the sheath impedance (-1500) suggesting that the voltage drop across the bulk plasma was typically small. The average electron density and temperature were found as described in Ref. (13). The purpose was to calculate the energy and flux of ions bombardment the substrate electrode as a function of reactor operating pressure and power input. These quantities have direct bearing on the etching rate and the degree of anisotropy of films.

Oxygen discharges are known to be electronegative, i.e., the negative ion density can exceed the electron density. However, for a pressure of 1 torr, Dettmer (23) found that the density of the dominant negative ion \((O^-)\) in a dc oxygen discharge became small, compared to the electron (and positive ion) density, for reduced electric fields in the bulk plasma \(E/N > 55\) Td \((1\) Td \(= 10^{-17}\) V cm\(^{-2}\)). The reduction of the effective electric field \((24) E/N)\) in the bulk plasma was calculated to be typically above 50 Td in the system studied. Under the assumption that a high frequency (and relatively high pressure) RF discharge of a given \(E/N)\) is comparable to a dc discharge with the same \(E/N)\), the negative ion concentration may therefore be neglected for the system at hand. However, since the present sheath model predicts the electron temperature \(T_e)\), the application of the model to an oxygen discharge should be limited to conditions under which the negative ion density is low.

The ion kinetic energy owing to the drift velocity is \(\varepsilon_i = 1/2 u_*^2\). By defining a dimensionless ion energy \(\varepsilon_i^* = 2\varepsilon_i/kT_e\) one finds by using \(u_*^2\) from Eq. [28]

\[
\varepsilon_i^* = \frac{(u_*)^2}{kT_e}
\]  

This is the ion energy in order of magnitude (associated with the ion drift velocity). However the total ion energy is higher since ions also have a random velocity component. It is the total ion energy which affects the etching kinetics. For the constant mean free path case considered here, Wannier (12) obtained a relationship between the ion kinetic energy in order of magnitude and the total average ion kinetic energy, which can be expressed here as

\[
\varepsilon_i^* = \frac{\varepsilon_i}{0.559}
\]  

One notes that a significant fraction of the total ion energy is associated with random motion. Equation [52] holds in the high pressure regime where the ion mean free path is much less than the sheath thickness, so that the ion velocity distribution function is well developed. Use of Eq. [52] under low pressure conditions will overestimate the total ion energy. The dimensionless ion flux is

\[
I_+ = \frac{\varepsilon_i}{\varepsilon_i^*} = \frac{1}{\sqrt{1 + 2Co}}
\]  

Quantities such as \(\varepsilon_i^*\) and \(I_+\) can be used in models for the ion-assisted kinetics to predict the ion-assisted etching rate. This can in turn be used in transport models of plasma reactors (13).

Figure 6 shows the dependence of ion bombardment energy on reactor pressure for different values of power input. Ion energies are indeed low. This is a result of the combination of low sheath voltages \((-200\) V) and many ion collisions in the sheath, at the relatively high pressures used. The situation may be drastically different in low power \((-10\) mtorr) reactive ion etching reactors with the wafer resting on the high sheath voltage smaller size electrode. Ion energies as high as 500 eV are easily obtainable in such systems. For a given power, ion energy decreases monotonically with pressure due to increased number of collisions and lowered sheath electric field. The decrease in energy with pressure may be partly counterbalanced by increasing the power input.

The variation of ion flux with pressure is shown in Fig. 7. The behavior can be explained by introducing dimensionless variables into Eq. [50] to obtain

\[
I_+ = \frac{n_+}{n_0} \left( \frac{kT_e/m}{1 + 2Co} \right)^{1/2}
\]  

The decrease of \(I_+\) with pressure is the result of \(T_e\) and \(n_0\) decreasing, and of Co increasing with pressure (note that \(n_0\) increases with pressure at low pressures, \(\leq 0.2\) torr in our case, but decreases at high pressures). The decrease of \(I_+\) is sharper at lower pressures mainly because \(n_0\) shows similar behavior with pressure (13). For constant pressure, ion flux increases almost linearly with increasing power. This is because \(n_0\) increases linearly with power and the term in parenthesis in Eq. [54] is only weakly dependent on ion. Ion flux is independent of sheath voltage for a sourceless sheath. This would not be the case if ionization occurred in the sheath by secondary electrons, for example. Such electrons are emitted from the electrode by the bombarding ions and are accelerated by the sheath electric field back into the glow. When pressure is high, secondary electrons can cause ionization in the sheath (10). This re-
results in higher ion current to the substrate and to thinner sheath as compared to the sourceless sheath case. When ionization in the sheath by secondary electrons takes place, the ion flux on the substrate electrode is

\[ I = \frac{I_{+o}}{1 - \gamma (\exp(Na,d) - 1)} \]  

where \( I_{+o} \) is the ion current injected into the sheath at the glow/sheath interface. The maximum ionization cross section for electrons in oxygen, for example, is \( \sigma_i = 2.72 \times 10^{-18} \) cm\(^2\). At previously, one may take neutral density \( N = 1.38 \times 10^{16} \) cm\(^{-3}\) (0.5 torr, 350 K) and \( d = 0.1 \) cm. For the ion energies of interest, the secondary electron emission coefficient has a value of \( \approx 0.1 \) for many electrode materials under noble ion bombardment, and is usually much smaller for molecular ion bombardment (11, 16). Then, for \( \gamma = 0.1 \), the denominator in Eq. [55] is about 0.95, which amounts to a 5% increase in current due to ionization in the sheath, under the above conditions. This is rather an upper limit on the expected degree of ionization by secondary electrons under these conditions. Similarly, ion-impact ionization may also be neglected. For example, using the rather high value of \( \sigma_i = 10^{-18} \) cm\(^2\) for the corresponding cross section and values of \( N \) and \( d \) as above, one obtains an ion multiplication factor of \( \exp(Na,d) = 1.14 \) ions produced per ion. Again, this is an overestimate of the ionization in the sheath, under the above conditions.

The dependence of ion energy on the sheath electric-field-to-pressure ratio, \( E/p \), has been recognized in the literature (1, 2, 12). In order to investigate further that relationship, \( E/p \) and ion energy (Eq. [32]) were calculated for different values of pressure and power in the range 0.3-2 torr and 20-200W, respectively. For each pressure-power pair the average sheath electric field was found by integrating the computed electric field distribution over the sheath thickness. The results are shown in Fig. 8. One observes that there exists a unique functional relation between ion bombardment energy and sheath \( E/p \). Low values of \( E/p \) occur at higher pressure (high Co). Under these conditions electrostatic and frictional forces balance each other, and Eq. [39] is applicable. Hence ion energy is linear in \( E/p \). At high \( E/p \) (low pressure), ion inertia is not negligible and the data deviate from the straight line passing through the origin. The deviation is negative because \( u_i \) \( \frac{ds}{dr} \) is positive, i.e. the ion velocity has not had a chance to achieve its "fully developed" value (ions strike the electrode before that happens).

No experimental data on ion bombardment energy and flux in oxygen plasma etching reactors could be found in the literature to test the predictions of the sheath model. Thompson et al. (1, 25), and Allen et al. (17) used a similar reactor system to that employed in the present study. Their experimental conditions were mostly within the range covered in the present work, except that the authors used gases such as CF\(_4\), C\(_2\)F\(_4\), Br\(_2\), and SF\(_6\). Values of the measured ion bombardment energy and ion flux were similar to the values shown in Fig. 6 and 7, although such similarity may be fortuitous. The variation of ion bombardment energy with power and pressure was also similar to that predicted by the present sheath model. Ion bombardment energy and flux critically affect etch anisotropy in plasma-assisted etching applications. A realistic plasma reactor model must provide for the calculation of these quantities.

**Summary and Conclusions**

An approximate model of the RF plasma sheath was developed. The ion drift velocity was found by coupling the ion "fluid" equations to Poissons' equation for the potential distribution in the sheath. Ion-neutral collisions were accounted for by a frictional force opposing the ion motion. An analysis of the collision dynamics showed that, for ion energies much greater than the background medium thermal energy, the frictional force may be taken proportional to the square of the ion velocity. The proportionality constant increases linearly with the collision cross section and with pressure. The dimensionless number Co (Eq. [35]) was found to be important in describing the sheath properties. Typical values of Co in high pressure (~1 torr) plasma etching reactors are around unity. For such values of Co, frictional and electrostatic forces on the ion quickly balance each other. An analytic solution was then derived yielding the ion bombardment energy as a function of the sheath potential. Such expression may be used for quick estimation of expected ion energy in high pressure plasma etching reactors. At the extreme of no collisions in the sheath (Co \( \to 0 \)) the Child-Langmuir law was recovered.

The model was applied to a high pressure parallel plate plasma reactor used to etch polymer films in an oxygen discharge. The energy of ions bombarding the grounded electrode was calculated to be only a few tens of eV, much lower than typical sheath voltages (~200V). Ion energy and ion flux decreased with pressure and increased with power. Over the range of variables investigated, the ion bombardment energy was found to be a unique function of the average sheath electric-field-to-pressure ratio, \( E/p \). In fact, ion energy was linear with \( E/p \) for values below about 2000 V/cm-torr, but deviated from linearity for higher \( E/p \) values.

The assumptions introduced in the model limit the parameter range over which the model is applicable. For ex-
ample, a time dependent phenomena (18, 19) must be incor-
porated for frequencies below 5 MHz (30). Ionization in
the sheath may become important for pressures higher
than about 1 torr (10). Finally, only the average ion energy
and not the ion energy distribution (8, 20) can be found.
Nevertheless, the simplified sheath model developed here
can be readily coupled to a more general plasma etching
reactor model, including transport and reaction phenom-
ena of the etching species, to calculate etch rate, uniform-
ity, and anistropy of etching thin films (13). The present
sheath model may also be used to construct equivalent
electrical Circuits of the sheath (21, 22). For example, the
sheath model may also be used to construct equivalent

**LIST OF SYMBOLS**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_0$</td>
<td>collision number, Eq. [35], dimensionless</td>
</tr>
<tr>
<td>$d$</td>
<td>sheath thickness, cm</td>
</tr>
<tr>
<td>$D$</td>
<td>collision diameter, cm</td>
</tr>
<tr>
<td>$E$</td>
<td>sheath electric field, V/cm</td>
</tr>
<tr>
<td>erf</td>
<td>error function</td>
</tr>
<tr>
<td>$F_i$</td>
<td>frictional force on ion, dynes</td>
</tr>
<tr>
<td>$g$</td>
<td>relative velocity of test ion with respect to neutral molecule, cm/s ($g =</td>
</tr>
<tr>
<td>$I_i$</td>
<td>ion current, mA/cm</td>
</tr>
<tr>
<td>$k$</td>
<td>Boltzmann’s constant, 1.38 $10^{-23}$ J/K</td>
</tr>
<tr>
<td>$m$</td>
<td>particle mass, g</td>
</tr>
<tr>
<td>$N$</td>
<td>neutral particle density, cm$^{-3}$</td>
</tr>
<tr>
<td>$n$</td>
<td>charged particle density, cm$^{-3}$</td>
</tr>
<tr>
<td>$p$</td>
<td>pressure, torr</td>
</tr>
<tr>
<td>$q$</td>
<td>fundamental charge, 1.609 $10^{-19}$ Cb</td>
</tr>
<tr>
<td>$T$</td>
<td>temperature, K</td>
</tr>
<tr>
<td>$t$</td>
<td>time, s</td>
</tr>
<tr>
<td>$u_i$</td>
<td>ion velocity, cm/s</td>
</tr>
<tr>
<td>$V$</td>
<td>potential, V</td>
</tr>
<tr>
<td>$v$</td>
<td>neutral particle velocity, cm/s</td>
</tr>
<tr>
<td>$x$</td>
<td>x-coordinate, cm</td>
</tr>
</tbody>
</table>

**Greek characters**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_i$</td>
<td>proportionality factor in Eq. [10], g/cm</td>
</tr>
</tbody>
</table>
| $\gamma$  | secondary electron emission coefficient, dimen-
|           | sionless                                        |
| $\varepsilon_i$ | ion energy in ordered motion, ergs or eV         |
| $\varepsilon_t$ | total ion energy, ergs or eV                     |
| $\varepsilon_\varepsilon$ | permittivity of the free space, 8.85 $10^{-14}$ F/cm |
| $\lambda_0$ | Debye length, cm                                 |
| $\sigma$  | collision cross section, cm$^2$                  |
| $\chi$    | scattering angle, degrees                        |

**Subscripts**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e$</td>
<td>electron</td>
</tr>
<tr>
<td>$el$</td>
<td>elastic collision</td>
</tr>
<tr>
<td>ex</td>
<td>charge exchange collision</td>
</tr>
<tr>
<td>i</td>
<td>ions, ionization</td>
</tr>
</tbody>
</table>

| p     | plasma                                           |
| s     | sheath                                           |
| t     | total                                           |

**Superscript**

* dimensionless quantity

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