SPE 84064

Self-Learning Reservoir Management

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Summary

In this work, we present an industrial automation framework for control and optimization of hydrocarbon-producing fields while satisfying business and physical constraints. The all-encompassing reservoir-management problem is decomposed into a hierarchy of decision-making problems at different time scales.

We exemplify the proposed approach through a case study on a multiple-layer reservoir with a classical waterflood problem, in which a numerical reservoir model is used as a virtual field. A model-predictive control (MPC) strategy is used to regulate well and field instrumentation at economically optimal set points determined by an overlying supervisory control level. The study demonstrates significant reduction in water-handling costs and increased oil recovery.

This work is a starting point for further development in automatic intelligent reservoir technologies, which capitalize on the abilities of permanent instrumented wells and remotely activated downhole completions.

Introduction

Reservoir management today is facing remarkable challenges in optimizing profitability while satisfying a number of constraints (physical, financial, geopolitical, and human). To optimize profitability, engineers traditionally have used mathematical models, field data, and domain expertise in an effort to make decisions about the best operating scenario. To increase the opportunities for profitability by greatly increasing the volume of available field data and the number of potential operating scenarios, the industry has recently started deploying sophisticated hardware for remote sensing and actuation of wells and facilities.

However, the acquisition of domain expertise about an oil field is a lengthy and often unstructured activity that cannot be undertaken easily on a continuous basis. In addition, because of the complexity and magnitude of an all-encompassing optimization problem for an entire oil field, decisions are made in a fragmented way for various pieces of that oil field. The lack of intelligent software applications exacerbates the situation. As a result, the capabilities of new sensing and actuation hardware have not been fully realized, making it difficult to justify the significant cost that such hardware imparts. In fact, it is fair to say that not much can be expected from a feedback-based decision-making loop unless all elements in the loop are properly configured, connected, and functioning. The industry state of the art is clearly far from this ideal end.

To address the above issues, we propose a fieldwide optimization and control framework[1] with the following key features:

- It uses a hierarchy of time scales to separate the levels over which decision making is performed, thus rendering a complex problem solvable.
- It integrates field data for continuous learning of key reservoir features, based on simplified empirical models suitable for real-time operations.
- It continuously optimizes reservoir performance while satisfying all business and physical (surface and subsurface) constraints.
- It uses an advanced feedback-control strategy, which can be implemented easily on field controllers at the wellhead or downhole.
- Its multiscale structure can naturally host optimization levels such as multilateral selection, well location, and portfolio optimization.

To exemplify the above framework, we develop in this work an MPC (receding horizon) scheme that underlies a supervisory optimization level, which predicts the best operating points of a hydrocarbon-producing field. The resulting structure is a self-learning and self-adaptive scheme that optimizes multiphase fluid migration in compartmentalized reservoirs while integrating downhole completions, wellhead restrictions, and business objectives and constraints.

To demonstrate the capabilities of the proposed approach, we develop a software prototype and test it on a case study using a commercial reservoir- and well-modeling environment as a virtual reservoir. Dynamic simulations show that the proposed strategy results in significant reduction of water injected and produced, with a simultaneous increase in overall oil recovery. For the case study presented, the self-learning reservoir-management strategy is able to reduce cumulative water production by almost 80% and reduce water injection by 55%, increasing project profitability from 13 to 55%.

In the sections that follow, we first give a background of current reservoir-management challenges as applied to continuous oilfield modeling and decision-making processes, multivariable optimization, and automatic control. We then present our proposed approach based on petroleum-system identification, an MPC strategy, and a closed-loop linear programming-optimization level that searches continuously for the best operating point of the field. Finally, we offer suggestions for further work.

Background

Reservoir Management. Reservoir management is the recurring process in which an oilfield operator uses mathematical modeling, data, and expertise to optimize reservoir profitability or some other stated objective about oilfield performance.

A common list of reservoir-management objectives includes the following:

- Decrease risk.
- Increase oil and gas production.
- Maximize recovery.
- Minimize capital expenditures.
- Minimize operating costs.
- Maximize profitability.

In managing reservoirs, operators are concerned about the surrounding environment (oil price, market access, available capital for expenditures), the limitations of the current infrastructure and physical properties of the subsurface/surface system, the confidence in the reservoir model upon which decisions are to be based, and the intrinsic risks of the operations.

Historically, reservoir management used to be identified with production engineering, and then it became synonymous with numerical reservoir simulation. It is now understood that reservoir management is an iterative process, of which numerical reservoir simulation and production engineering are only two components. A methodology is gradually emerging to facilitate the routine implementation of reservoir management. We hope that this work can contribute to that goal.

Current Reservoir-Management Challenges. The current industry trend is toward developing even more risky complex reservoirs with lower expected profit margins. Additionally, success rates are becoming less exciting, capital expenditures generally exceed budget, operating expenditures are commonly well above initial projections, first oil is 1 to 2 years late, production rates are only a fraction of those projected,[2] water breakthrough occurs earlier than expected, and injection rates are lower than anticipated. The end result is unattractive project economics.

To improve project economics, the operator has to search for key reservoir knowledge and adaptation of new technologies. Unfortunately, knowledge of the reservoir is gained only through experience and field production. This is often a tedious exercise, especially when no proper data are collected or analyzed, when reservoir models are not updated continually, and when there is very little ability for remediation actions.

Recent Advances Relevant to Reservoir Management. During the last two decades, the oil industry has started focusing on cost and risk reduction. In that sense, highly impacting technologies (3D seismic, horizontal wells, automatic control) have been adapted from other disciplines and industries (computing, medicine, petrochemical, and military) and evolved further. A selection of such technologies includes the following.

Reservoir Characterization. It has moved from paper and pencil to highly sophisticated, large-scale visualization rooms in which cross-disciplinary teams model and simulate the reservoir physics and design the facilities that best accommodate the exploitation needs. Numerical reservoir simulation, the most popular forecasting technique, is possible through an inverse modeling technique in which reservoir parameters are set to match the reservoir production history. Further human- and computer-intensive efforts are required to match a nonunique solution, which is usually conditioned to a finite uncertainty quantification of multiple geostatistical realizations.

Poor performance prediction from existing models and lack of integration with continuous data acquisition are considerable incentives for developing effective modeling strategies that incorporate knowledge of the reservoir and its facilities during its whole producing life.

Multivariable Optimization in the Oil Industry. Multivariable optimization techniques have been used in many ways in the oil industry to support decisions, attempting to accomplish daily tasks such as resource scheduling, optimum history matching of reservoir parameters, optimal facilities design,[3] optimal well location and trajectory,[4] production parameter settings,[5,6] and optimization of the displacement efficiency or recovery factor.[7] Such optimization uses models that are generated offline, either by using first principles or semiempirically by using data acquired in the field.

Despite having been used extensively in the petrochemical industry, multivariable optimization has not fully penetrated the oil industry. To the extent used, it lacks the connection with the real field and does not consider the inclusion of dynamic data for continuous updating of models, as is mostly the case in downstream applications.

A Continuous Trend Toward Smart Fields. Lately, the industry has started deploying downhole and surface-measurement and -control (remotely operated valves) capabilities to automate many of the tasks that are carried out manually today. Remotely activated well-flow control devices are intended to avoid future well intervention while efficiently controlling injection or production of multiphase fluids from different pay zones. These "smart" technologies are intended to enable:

- Remote adjustment of the well's operating point.
- Continuous and automatic fine-tuning of production.
- Improvement of project economics.

Additionally, to produce such results, it is necessary that proper information is gathered and integrated with reservoir models and petroleum production-engineering tools from the wellbore and production facilities. These will enhance the reservoir image of the rock- and fluid-properties distribution (carried out on geologist platforms) and the interaction with the surface infrastructure.

On the other hand, these technologies increase the well cost and the overall infrastructure complexity. Because of that, their economic value has to be fully justified before they are implemented.[8] Certainly, such technologies are not a panacea. However, they may be the only solution for well-identified candidates such as deep offshore developments.[9]

The lack of integration and automation software tools has also contributed to low excitement for smart completions. Much has been said about the smartness of fields and reservoirs, but few have attempted to define what this intelligence is about and how a hydrocarbon field can run in an autonomous and unmanned way.

Automatic Process Control in the Oil Industry. There have been a number of automatic control applications in the oil industry. Since the early days of automation, instruments have been deployed in the oil field to control basic process variables (pressure, flow, temperature) at the regulatory level and to tune up process operating points (gas-lift allocation, production levels) at the supervisory and optimization level.[6,10–14]

Control technology has been of particular interest for optimizing reservoir performance and displacement efficiency (namely, percentage of oil recovery per unit volume). Several authors[5,7,15,16] have proposed optimal control-theory strategies[17] for enhancing oil recovery in steam-, CO_2-, gas-, and water-injection projects. In many of these strategies, a control variable is manipulated while an objective function is optimized subject to a number of constraints. A feature of these previous efforts is that all of them were devised for offline control, essentially to optimize field parameters only once. Many of these strategies were not even considered to be deployed in oilfield controllers.

Proposed Approach

The Paradigm of Field Operations Hierarchy. Our vision for oilfield management is captured in **Fig. 1,** which presents a multilevel industrial automation hierarchy as it could be applied to the oil industry.[18] Each level passes down the results of decision making as goals for lower levels, while lower levels provide data to upper levels for use in feedback-based decision making. The frequency of decision making increases as ones goes to lower levels. Although lower levels of the operations hierarchy involve the manipulation of critical variables for the correct functioning of field parameters, the operating goals of the upper levels are also responsible for the value creation of the project. For example, the right orchestration of gas allocation among several wells for gas lift is as important as the regulatory control implementation of these set points at the well level (i.e., the closed-loop control between the gas valve and the produced mass).

This decomposition of the overall operations task into a number of problems at multiple levels lowers complexity and makes the overall problem manageable.

Focus of This Work. In the context of the hierarchy presented in Fig. 1, this work will focus on demonstrating how model-based optimal decision making can be developed at different time scales (from days to months) corresponding to different levels of the hierarchy in Fig. 1.

A novel computer-aided engineering tool (model-based decision-making engine) is proposed for enhancing reservoir understanding and performance as field data are collected. This tool combines advanced process control with data-driven lumped parametric modeling that honors reservoir physics. The model-based decision-making engine is intended to manipulate remotely operated valves on downhole completion and wellhead instrumentation.

The capabilities of the proposed engine encompass:

- Online identification of reservoir and well models.
- Use of these models for online multilevel and multiscale optimization and self-tuning control.

At the regulatory control level, an MPC (receding horizon) scheme controls the process to its set point by solving an online optimization problem. The MPC uses a dynamic model that is updated continuously from available data over time.

The MPC level underlies a supervisory optimization level, which predicts the best set points of the hydrocarbon-producing field based also on the same dynamic model and the maximization of the net present value (NPV) objective function.

The regulatory (MPC) level is related to short-term optimization of well deliverability, in contrast to the supervisory level, which performs long-term optimization.

The resulting structure is a self-learning and self-adaptive scheme that optimizes multiphase fluid migration in compartmentalized reservoirs while integrating downhole completions, wellhead restrictions, and business constraints. We call this structure "self-learning reservoir management."[1]

The concepts of self-optimizing control and self-tuning regulators have been around for many years.[19] To the best of our knowledge, this work is the first attempt to develop similar concepts for optimizing reservoir performance.

Feasibility of the Proposed Approach. The ideas of multilevel optimization discussed in the preceding section have been applied successfully in downstream petrochemical industries. Some elements of the individual levels have also been explored in the upstream industry, as discussed below.

Extensive Experience in Process Identification. Online identification of process models, referring to the use of regression for the approximation of process behavior based on data, has been used extensively in petrochemical industries since the 1970s.[20,21]

Several authors[22,23] have attempted to model oil reservoir and producing field properties using data-driven models (e.g., neural networks and fuzzy logic). Renard[24] proposed an identification structure for predicting the fractional flow of water and used principal-component analysis to identify crosswell interference.

The main characteristic of these previous efforts is that all models were conceived for offline identification, basically to describe processes that will not vary with time, such as reservoir-property distribution, lithology, electrical response, and multiphase flow in pipes. Reservoir production and pressure response is highly nonlinear with time-varying characteristics. Nonlinear modeling may be complex and difficult to implement, but it is not always necessary.[25–27] For this problem, we show in the sequel that a model linear in the parameters can adequately capture the dynamic behavior of a reservoir, thus making parameter identification easy by means of linear regression.

Experience With MPC. MPC,[28] a technique in which the error between the predicted plant response and the set point is minimized online over a receding horizon and subject to constraints, has been used for many years in the petrochemical and food industries.[29] The capabilities of MPC have been extended to meet the requirements of a robust strategy for simultaneous identification and control of linear and nonlinear systems.[30–33]

Benefits of the Proposed Approach. Individual optimization problems in the upstream oil industry come in a variety of forms, for which various optimization algorithms have been proposed and used. This work does not purport to introduce a new optimization algorithm. Rather, it emphasizes the importance of formulating the optimization problem as a multilevel problem so that the overwhelming initial complexity can be reduced, and the resulting problem can be manageable. In particular, the case study we present in this work requires standard linear and quadratic programming tools that are readily available. As additional optimization levels are included in the formulation, additional optimization paradigms may be considered as necessary.

The main benefit of this work is to provide a self-adjusting tool that orchestrates live data for enhancing reservoir understanding and economic performance. As field data are collected over time, the computer-aided engine "self-learns" the reservoir dynamics and continually produces "smart" actions for control hardware in the field by performing dynamic optimization that satisfies short- and long-term project objectives and constraints.

The proposed approach provides a framework for exploiting the significant capabilities of smart downhole completions and wellhead instrumentation, which otherwise might remain underused after their implementation. In addition, by realizing the economic benefits, the proposed approach can offer rational justification for the significant capital expense required for such hardware. Widespread application of this approach can have far-reaching implications for the entire oil and gas industry in terms of lowering production costs and increasing project profitability.

Oil Field Viewed as a Dynamic System. A system is a structure in which variables of different kinds interact and produce observable output signals by the action of external stimuli: manipulated inputs and disturbances. In this context, the petroleum system **(Fig. 2)** is a dynamic structure of many observable outputs (fluid rates, concentrations, and pressures) that respond to the action of external stimuli (flow choke settings, injection rates, separator pressure, and artificial-lift quantities), measurable (pipe flow constraints) and unmeasurable disturbances, and model uncertainties (reservoir heterogeneity and reservoir-fluid distribution).

Data-Driven Modeling and Identification. The term data-driven identification refers to the use of actual data to fit the parameters of a mathematical model. It uses statistical regression analysis and has recently evolved to encompass various linear and nonlinear model structures. Dynamic systems are often represented in the discrete-time domain, particularly when observed data are collected by discrete sampling. In most of this paper, the sampling interval is assumed to be one time unit.

Parametric Fluid-Flow Modeling. Mathematical models can be based on first principles (e.g., conservation of momentum, mass, and energy), empiricism, or a combination thereof. First-principle models can be combined with constitutive equations to generate model structures that are valid over a wide range of operating conditions. However, they may be cumbersome to develop and manipulate. Empirical model structures, on the other hand, may be easy to develop but may not be as accurate and cannot easily be used outside the range of data used to fit the model parameters. Hybrid models may use a first-principle structure along with empirical constitutive equations (e.g., Darcy’s law, ideal gas law) and may rely on data to identify values of model parameters. Because of this, hybrid models are often easier to develop and manipulate than raw first-principle models while maintaining model fidelity outside the range of the data used for model-parameter identification.

The process by which one uses data to identify values of the parameters of an empirical or hybrid model is referred to as learning. We call self-learning the process by which a system uses its past operating data to learn a related model. The term learning has been used extensively in conjunction with neural networks to denote their ability to "learn arbitrarily complex nonlinear mappings from available data."[34,35] Even though the range of mappings that neural networks can approximate is extremely wide, learning is not a capability unique to neural networks, but one that is possessed by every parametric model structure, including standard linear model structures.[18] When data are noisy, as is always the case in practice, one has to balance the accuracy of data fitting with the model's predictive ability by appropriate selection of model structure. Various quantitative criteria can be used to that end (e.g., the related cross validation[36] or Akaike[37] information criteria, which balance the fitting and predictive abilities of a model).

For the above reasons, we propose to use hybrid models for self-learning reservoir management. Our aim is to reduce the brittleness of purely empirical (black-box) parametric models (such as neural networks) when extrapolating beyond the training data by forcing conformance to physical laws, and to improve performance in the range of the training data.

Hence, the first step toward developing a model structure for a hydrocarbon-production system is to assign input and output variables and to recognize the interdependence of these variables while honoring the physics behind the process.

Continuously acquired field data can be used to predict reservoir performance through linear relationships such as

$$\begin{bmatrix} q_o^k & q_w^k & q_g^k \end{bmatrix} = \begin{bmatrix} 1 & \overline{p}^k & p_{wf}^k & \left(p_{wf}^k\right)^2 \end{bmatrix} \begin{bmatrix} a_0 & b_0 & c_0 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix} \quad \text{.............(1)}$$

and

$$\left[\left(\overline{p}\right)^k\right] = \begin{bmatrix} d_0 & d_1 & d_2 & d_3 & d_4 & d_5 \end{bmatrix} \begin{bmatrix} 1 & \overline{p}^{k-1} & q_o^{k-1} & q_w^{k-1} & q_g^{k-1} & q_{wi}^{k-1} \end{bmatrix}^T,$$

$$\text{.....................................(2)}$$

where the right sides of Eqs. 1 and 2 contain the information data at a certain instant and parameter matrices that fit input to output values. The nomenclature and derivation of Eqs. 1 and 2 can be found in Appendix A, which describes modeling of fluid rates and reservoir pressure using simple analytical equations in conjunction with Vogel's and Fetkovich's empirical equations.

Because those models capture the main and simplified physics features (i.e., flow rate is a parametric combination of current flowing pressure; pressure is a parametric combination of mass flow rates), they can be used in a number of field cases in which the variables to be estimated are parametric combinations of the measured variables. For example, flow rates for the field depicted in **Fig. 3** are calculated with Eqs. 1 and 2. Flow rates can be calculated individually for each layer or as the summation of fluid rates in both layers. Average pressures can be calculated as the gross-rock-volume-averaged pressure from all layers or as individual average reservoir pressures for each individual layer.

The simplified average pressure response may differ from the actual one up to a constant value. This difference can be adaptively eliminated within a few iterations of the regression process. Therefore, the approximation is adequately absorbed by the identification coefficients.

For future applications (e.g., different number of layers, wells, heterogeneous distribution of rock properties), a new set of system-identification equations may be derived easily on the basis of the required response and measured variables, as long as the variables to be estimated are still parametric combinations of the available measured variables.

Control and Optimization for Reservoir Management. Well fluid-rate control has been related to optimization of fluid displacements in porous media.[38,39] With the onset of smart wells, there has been an increasing need for development of control applications that will actuate available hardware. There have been attempts to model downhole valves and segmented well architectures[40] and to create optimization control routines[4,15,16] that will automatically adjust valve settings to optimize well and field throughput.[41–43]

Some advanced-recovery processes and novel architectures also have been subject to control and optimization treatment. Queipo *et al.*[44] suggested a surrogate modeling-based approach for the optimization of steam-assisted gravity drainage (SAGD) processes. Yeten *et al.*[4] suggested the optimization of location and trajectory of nonconventional wells.

MPC Level. MPC is a class of computer control algorithms that explicitly uses a process model for online prediction of future plant behavior and computation of appropriate control action through online optimization of a cost objective over a future horizon, subject to various constraints.[29]

Consider the injector/producer problem in Fig. 3 in a two-layer reservoir, with data found in Appendix C. Layers 1 and 2 are the producing units with distinctive (e.g., ratio 10:1) *kh* values. There is no vertical communication between the layers, and vertical permeability values are irrelevant for this example.

In Appendix B, we show the development of the basic elements of an MPC structure that includes simultaneous parametric identification of a reservoir model, as shown in **Fig. 4.** The structure of Fig. 4 includes an identification loop (upper part) and a feedback control loop (lower part). The identification loop identifies a model as structured input/output relationships obtained by reduced-rank regression methods [e.g., partial least squares (PLS)] on equations such as Eqs. 1 and 2. This model is used by the lower part of Fig. 4, where an MPC controller implements process inputs for the next timestep, based on the result of an optimization that minimizes the error between the set point and the actual process output as well as the required control action.

Fig. 5 shows an example of the structure of Fig. 4 in action, where the injector/producer problem of Fig. 3 is tested. In the upper-left plot, the average reservoir pressure's set point is 10,000 psia and is kept within some constraints by the "aggressive" manipulation of the injector input (middle-right).

Oil production (upper-middle of Fig. 5) follows a general exponential decline behavior; at t=80 days, the production setpoint for each layer is changed to 5,000 STB/D without any further changes. Bottomhole pressure (lower-left of Fig. 5) in both layers is continuously adjusted to meet the set points. Because Layer 1 has better reservoir properties (i.e., larger *kh*), the behavior of the bottomhole pressure is always above the one for Layer 2 to meet the same target.

The change in oil-production set point (at t=80 days) is captured as a disturbance in the injector. After a short period, the injector is back on injection and saturates its input to 15,000 B/D. After 150 days, reservoir pressure increases toward the upper limit.

It can be seen in Fig. 4 that bottomhole pressures (inputs) in both layers were continuously adjusted to meet the set points. Without prior knowledge of reservoir characteristics and based only on operating data, the behavior of the bottomhole pressure was always higher for larger *kh* than for smaller *kh* to meet the appropriate target.

NPV Optimization Level. In the context of the field operations hierarchy of Fig. 1, an *upper optimization level* optimizes an NPV objective function subject to current reservoir model and physical constraints **(Fig. 6).** The NPV optimization exercise reduces to

$$\max\left\{NPV^k\right\} = \underbrace{\max}_{u_1^k, u_2^k, u_3^k} \left\{\sum_{k=1}^{N} au_1^k + bu_2^k + cu_3^k + d\right\}, \quad \ldots(3)$$

Where u_1, u_2, u_3 are the decision variables corresponding to the flow settings of Layers 1 and 2 and the water-injection rate, respectively, and a, b, c, and d are the resulting coefficients after combining locally linearized versions of the models of Eqs. 1 and 2 with Eq. D-1, as explained in Appendix D.

The constraints are imposed by the reservoir, wells, surface equipment, cost, and schedule. For example, in finding the solution for optimizing the deliverability of a well, the physical constraints are given by the reservoir productivity and the tubing performance, as shown in Fig. 6, where the shaded area denotes the polytope of feasible solutions.

A linear programming-optimization routine was used to find the optimum solution. The solution vector is the set of operating parameters that optimizes reservoir performance and value. This set of operating parameters is the set point of the MPC level that underlies this optimization level.

The above optimization exercise is carried on with the information available at every timestep, assuming that future reservoir behavior is described by the current model. This assumption is known in optimization and control theory as the certainty equivalence principle. In subsequent timesteps, that model is going to be updated, and the NPV will be refined continuously.

Self-Learning Reservoir Management. A self-learning reservoir-management strategy can be achieved by combining the following three elements, discussed in the preceding sections:

- Parametric fluid-flow modeling.
- MPC.
- NPV optimization.

Fig. 7 shows the structure resulting from interconnection of these three elements. As shown in Fig. 7, the captured model at the regulatory level feeds a reservoir-performance forecast block, which generates the fluid-flow functions to be used in the NPV objective function. Optimization of the NPV objective function produces the set points that are fed to the MPC level.

Case Studies

Example: Data-Driven Reservoir-Performance Prediction. Consider the reservoir example with the data in Appendix C, and using only data for Layer 1. The decision-making variables (inputs) are u_1 and u_2, and the controlled variables (outputs) of that system are y_1 through y_6, as shown in **Fig. 8.** The input variable u_3 is not used in this example.

To test the identification procedure, an algorithm was developed to identify the relationships between inputs and outputs in the context of Eqs. 1 and 2. The identification algorithm considers the last 30 days of history and produces a model used to make predictions for the next 30 days. The results are shown in **Fig. 9.** Almost perfect agreement is observed between the average reservoir pressure (y_1) calculated by the simulator (Ym) and the one calculated by Eq. 2 (Ycalc), as depicted in the upper left chart of Fig. 9. A very good forward prediction of the dynamics is also observed. Oil-rate fitting and prediction can be seen in the upper middle of Fig. 9, for which an exponential decline curve (red line) is also shown for comparison. The water-production rate is presented in the upper left of Fig. 9.

Although the model cannot predict the onset of water before water has broken through, it is progressively adapted to the new wellbore conditions (i.e., it is transformed to represent a water increase). A minor deviation in the prediction of water production can be observed (upper right of Fig. 9), although the trend is preserved. The prediction of sudden changes in water-injection rate (middle right of Fig. 9) was less accurate. However, the main features were captured.

As shown in Fig. 9, pressure-prediction errors were negligible and randomly distributed. Errors observed in the water rate and water cut before 130 days (upper-right and middle-left charts in Fig. 9)—i.e., unfeasible negative water rates—arose because water had not reached the production well, and the measurement from the simulator was erratically perturbed with numerical errors.

Fig. 10 shows the evolution with time of the identified parameters. One could appreciate the great variation of each coefficient through time (i.e., the adaptability of the model to "learn" the system's variations). However, this also is an indication of the difficulty in predicting future performance for large horizons and the need to consider multiple scales.

Example: Self-Learning Reservoir Management. A five-spot waterflooding pattern is studied to understand the behavior of the self-learning reservoir-management strategy under the complexity of a fieldwide production. A multilayer reservoir with five distinct *kh* characteristics is used. Rock and fluid are described in Appendix C. The five-spot waterflooding problem is exposed to both noncontrolled and controlled (self-learning) scenarios.

In this example, the input variables (u_1, u_2,…,u_{25}) are the bottomhole pressures [$p_{wf,ij}$] of every layer for all production wells, and the output variables are the oil and water flow rates [$q_{o,i}$ and $q_{w,i}$] at every production well and the total flow rate [$q_{w,i}$] for the water-injector well.

Figs. 11 through 16 show the simulation results under the self-learning reservoir-management strategy and base noncontrolled case. The distribution of fluids (oil saturation) is shown at the end of the simulation (2,200 days).

The noncontrolled case (Fig. 11) showed early water breakthrough in Layer 1 (upper layer of high permeability), which reduced the ability of the well to flow under current vertical-lift performance constraints. The ultimate recovery is impaired because of the inability of the well to lift high-water-cut flow rates.

The simulation under the self-learning reservoir-management strategy (Fig. 12) shows a more uniform distribution than the one under no control (Fig. 11); water breakthrough was detected and controlled, and bottomhole pressure inputs in the watered areas were reduced up to full zone shutoff. This permitted better vertical lift on the well. For the same period of time, recovery was accelerated at a minimum effort.

Although the vertical sweep efficiency looks better for the self-learning reservoir-management case, both cross sections (Figs. 11 and 12) show similar fluid distribution (i.e., there are no dramatic differences).

However, it is noticeable that water breakthrough in Layers 1 and 2 is indeed delayed for the self-learning reservoir-management case (Fig. 12). In the following graphs (Figs. 13 through 15), flow rates and cumulative flow rates are compared for each case.

Oil flow rate and cumulative flow rates (Figs. 13 and 14) look slightly better for the self-learning reservoir-management case (i.e., there are no dramatic differences, only a 5% increase in oil recovery, equivalent to approximately U.S.$5 million over the project life).

However, a more remarkable result is shown for the water produced and injected. As water breaks through (t=50 days) in the high-permeability layer, it is detected and controlled in the self-learning mode (Fig. 15), where the simulation shows the continuous control and adaptation to better performance. As more water is produced from flooded layers, automatic regulatory action will reduce the contribution from them until shutoff while meeting business objectives (water-handling cost). Therefore, water rate was minimized for the self-learning case.

In the noncontrolled case, water injection is not guided by any economic objective. Rather, both water-injection zones are fully open and respond to the reservoir-pressure decline driven by production.

As a comparative result, the self-learning case was able to reduce cumulative water production by almost 80% (Fig. 16) and reduce water injection by 55%. With an average price of U.S.$2.5/bbl of water-handling costs, either for compression or treatment, this rounds up to an additional project value of $92.5 million over a period of 2,200 days.

Discussion and Suggestions for Further Study

Continuous Feedback Adjustment. Tracking of set points of reservoir pressure and production and injection rates (outputs) was affected by online adjusting of the flow settings (valve openings) of injectors and producers (inputs) in a feedback fashion. This was made possible by having an adequate dynamic model that could predict the effect of inputs on controlled outputs. This model was updated continuously and used by the feedback controller (MPC), which was fed with optimal set points by an upper NPV optimization level. This approach is different from previous approaches found in literature, in which optimization was attempted off-line in a fragmented fashion over a single time scale.

Disturbance Rejection. The MPC strategy was able to reject disturbances such as bottomhole-pressure changes in adjacent completions or pressure changes from the injector.

Number of Sensors and Actuators. The dependence of the ultimate outcome of the self-learning reservoir-management strategy on the number and location of sensors and actuators was clearly demonstrated. By comparing various alternatives of increasing sensing and actuating capability, the current framework can help to determine the optimal number and location of sensors and actuators required to implement an effective reservoir-management strategy.

Limited Optimization Ability. Once the physical constraints were reached (minimum bottomhole and wellhead pressure), the controller role was only to monitor the reservoir response, and no further inputs could be implemented. Thus, the system was not able to improve beyond this point. This creates an opportunity to interface the proposed strategy with an upper level of optimization that would select a different range of constraints or resizing of the system.

Persistence of Excitation. No controller degradation was observed because identification was performed in a closed loop. Model-process mismatch was acceptable for control purposes. Proper input excitation may be required for guaranteeing a well-conditioned information matrix during identification. If necessary, persistent excitation also may be embedded into the MPC algorithm, and the set-point parameter would be changed by the allowable flow constraints. Remotely operated actuators (smart-well completions and automated wellheads) will permit controlled continual excitation so that enough information is captured for identification.

Fault-Tolerant Controller. Further strategies may be added to the system to counter possible sensor or actuator faults. System redesign also may be suggested for enhanced fault tolerance (e.g., by hardware redundancy or virtual sensors).

Modeling. The volumetric average reservoir pressure was assumed to be the drainage-area boundary pressure. This could lead to some biased errors for small, confined compartments. Furthermore, a more detailed model may help to reduce large variations in identified coefficients.

In general, it is important to develop empirical model structures that do not violate first principles yet have parameters that can be identified easily in real time from field data. While such models need not be perfect, they must correctly capture elements of the reservoir's dynamic behavior that are essential for continuous optimization using feedback.

Sampling Rates and System Time Constants. Depending on the data-sampling rate, it is possible to capture reservoir pressure-transient behavior and, hence, characterize properties as in traditional well-testing analyses.

Cost of Computations. Online MPC with simultaneous identification was enabled by intense computing calculations for every timestep k. For a finite impulse response (FIR) model of order n=5, the number of computations for obtaining the regression vector were on the order of $(n \times nU)^3 + (N)^2 \approx 200 \times 10^3$ operations, where n is the number of regression coefficients; there were nine inputs (nU) and 6 outputs (nY); N=400 is the number of measurements. The MPC algorithm performs an operation of the form $\mathbf{u_F}=\mathbf{B}^{-1}\mathbf{A}$, where $\mathbf{B}$ is on the order of $(n \times nY)$. Therefore, the computations are on the order of $(n \times nY)$= 200×10^3 operations, for n=10. For a 2-GHz Pentium 4 PC, every timestep took approximately 5 seconds, which is clearly suitable for field implementation.

Conclusions

A novel multilevel self-adaptive reservoir-management strategy has been developed. It entails two optimization levels: the *upper level* optimizes the NPV function subject to current reservoir-model and physical constraints by selecting optimal values for input variables such as production and injection flow settings. The upper level passes these optimal values as set points to the lower level, which uses model-based predictive control to ensure that controlled variables follow their set points. This strategy is part of general field-operations hierarchy. Three-phase fluid migration in a multilayered reservoir can be optimized continuously using the above strategy. The proposed approach represents a distinct departure from previous ones, in that:

1. It uses multilevel optimization.
2. It is data-driven.
3. It is suitable for real-time operations.
4. It continuously optimizes reservoir performance while satisfying all business and physical surface and subsurface constraints.
5. It uses feedback corrections.
6. It provides a framework for integration and reconciliation of diverse reservoir-management objectives.

We recommend that a test of the proposed approach be attempted with a real field to validate the outcomes of this research.

Nomenclature

a_0 = oil-rate parametric coefficient, STB/D

a_1, a_2= oil-rate parametric coefficient, STB/D/psi

a_3 = oil-rate parametric coefficient, STB/D/psi^2

b_0 = water-rate parametric coefficient, STB/D

b_1, b_2= water-rate parametric coefficient, STB/D/psi

b_3 = water-rate parametric coefficient, STB/D/psi^2

c_0 = gas-rate parametric coefficient, Mscf/D (notice that this is c sub zero, and should not be confused with oil compressibility as c sub “o” for oil)

c_1, c_2= gas-rate parametric coefficient, Mscf/D/psi

c_3 = gas-rate parametric coefficient, Mscf/D/psi^2

$C_F^{\,k}$ = total fixed costs (overhead, leases, capital cost) at time interval k

C_{wi} = cost of treatment and compression of injected water per unit barrel, U.S.$/STB

C_{wp} = cost of treatment and disposal of produced water per unit barrel, U.S.$/STB

d_0 = pressure parametric coefficient, psi

d_1, d_2, d_4 = pressure parametric coefficient, dimensionless

d_3, d_5 = pressure parametric coefficient, 1/psi

ΔT_k = size in days of the time interval

i = annual discount factor

i_1, i_2 = Injector 1, Injector 2

I_T^k = total capital investment on field assets (wells, surface facilities) at time interval k

k = time-interval number

n = number of regression coefficients

N = number of measurements or intervals

N_p = cumulative produced oil, STB

nU = number of inputs

nY = number of outputs

p_1, p_2 = Producer 1, Producer 2

P_o = net selling revenues of oil, U.S.$/STB

P_g = net selling revenues of gas U.S.$/scf

q_{iw} = water-injection rate, STB/D

q^k = flow rate at time k, STB/D

$q_{p,sp}$ = set point of flow rate of phase p, STB/D

q_{pi} = flow rate of phase p, well i, STB/D

q_p^k = daily production of oil (STB/D), water (STB/D), and gas (scf/D) at time interval k

q_{wi}^k = daily injection of water (STB/D) at time interval k

r^k = tax rate at time interval k

$\mathbf{u_F}$ = future moves of the input variable vector

u_i = input variable number i

W_e = cumulative water injected, STB

W_p = cumulative water production, STB

y_i = output variable number i

WWPR = well water production rate, STB/D

WWCT = well water cut, fraction

WWIR = well water injection rate, STB/D

Acknowledgments

The authors gratefully acknowledge the U. of Houston Cullen College of Engineering Research Foundation and PDVSA for funding this research.

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Appendix A—Parametric Porous-Media Modeling

Fluid Rates. Oil, water, and gas rates are linearly related to a pressure-integral function, which declines monotonically as time passes, or simply a linear relation between average reservoir pressure and the well flowing pressure, p_{wf}, as

$$q_o(t) \propto \frac{kh}{p_D}\int_{p_{wf}}^{\overline{P}} \frac{k_{ro}}{\mu_o \beta_o} \cong f_L\left(p_{wf}, \overline{P}\right), \quad \text{.............................(A-1)}$$

$$q_w(t) \propto \frac{kh}{p_D}\int_{p_{wf}}^{\overline{P}} \frac{k_{rw}}{\mu_w \beta_w} \cong f_L\left(p_{wf}, \overline{P}\right), \quad \text{.............................(A-2)}$$

and

$$q_g(t) \propto \frac{B_g\left(GOR - R_s\right)kh}{p_D}\int_{p_{wf}}^{\overline{P}} \frac{k_{ro}}{\mu_o B_o} \cong f_L\left(p_{wf}, \overline{P}\right), \quad \text{.(A-3)}$$

where k is the absolute permeability (md), h is the pay thickness (ft), k_{rp} is the relative permeability of phase p (fraction), β_p is the formation volume factor of phase p (res bbl/STB) or (res bbl/scf), μ_p is the viscosity of phase p (cp), GOR is the producing gas/oil ratio, and p_D=ln(r_e/r_w) at steady-state or p_D=ln(0.472r_e/r_w) at pseudosteady-state conditions.

Without loss of generality at any particular time, oil, water, and gas flows can be represented, in oilfield units, as

$$q_o = \frac{kk_{ro}h\left(p_e - p_{wf}\right)_o}{141.2B_o\mu_o\left(p_D + s\right)}, \quad \text{...(A-4)}$$

$$q_w = \frac{kk_{rw}h\left(p_e - p_{wf}\right)_w}{141.2B_w\mu_w\left(p_D + s\right)}, \quad \text{...(A-5)}$$

for which flow functions can simply be modeled as

$$q_o^k = a_1 \cdot \overline{p}^k + a_2 \cdot p_{wf}^k \,. \quad \text{..(A-6)}$$

$$q_w^k = b_1 \cdot \overline{p}^k + b_2 \cdot p_{wf}^k \,. \quad \text{...(A-7)}$$

$$q_g^k = c_0 \cdot q_o^k \,. \quad \text{..(A-8)}$$

Recall that Vogel's backpressure and Fetkovich's equation for flow above and below the bubblepoint pressure are

$$q_o = J^*\left(\overline{p} - p_b\right) + \frac{p_b \cdot J^*}{1.8}\left[1 - 0.2\left(\frac{p_{wf}}{p_b}\right) - 0.8\left(\frac{p_{wf}}{p_b}\right)^2\right] \text{... (A-9)}$$

$$q_o(t) = J^*\left(\overline{p} - p_b\right) + \frac{J^*}{2 \cdot p_b}\left[{p_b}^2 - {p_{wf}}^2 \right] \quad \text{....................(A-10)}$$

Therefore, oil, water, and gas flow can be modeled as

$$q_o^k = a_0 + a_1 \cdot \overline{p}^k + a_2 \cdot p_{wf}^k + a_3 \cdot \left(p_{wf}^k\right)^2, \quad \text{....................(A-11)}$$

$$q_w^k = b_0 + b_1 \cdot \overline{p}^k + b_2 \cdot p_{wf}^k + b_3 \cdot \left(p_{wf}^k\right)^2, \quad \text{....................(A-12)}$$

$$q_g^k = c_0 + c_1 \cdot \overline{p}^k + c_2 \cdot p_{wf}^k + c_3 \cdot \left(p_{wf}^k\right)^2, \quad \text{....................(A-13)}$$

and the values of $a_0,\ldots,a_3$, $b_0,\ldots,b_3$, $c_0,\ldots,c_3$ are the parameters to be fitted through regression. The equivalent matrix form is

$$\begin{bmatrix} q_o^k & q_w^k & q_g^k \end{bmatrix} = \begin{bmatrix} 1 & \overline{p}^k & p_{wf}^k & \left(p_{wf}^k\right)^2 \end{bmatrix} \begin{bmatrix} a_0 & b_0 & c_0 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}. \quad \text{...(A-14)}$$

Because measurements are taken continuously over time, a least-squares estimator can be used to approximate an optimum fitting parameter vector that best fits the experimental data, or any other technique, such as PLS, neural networks, or subspace identification.

$$\begin{bmatrix} q_o^k & q_w^k & q_g^k \\ q_o^{k+1} & q_w^{k+1} & q_g^{k+1} \\ \vdots & \vdots & \vdots \\ q_o^N & q_w^N & q_g^N \end{bmatrix} = \begin{bmatrix} 1 & \overline{p}^k & p_{wf}^k & \left(p_{wf}^k\right)^2 \\ 1 & \overline{p}^{k+1} & p_{wf}^{k+1} & \left(p_{wf}^k\right)^2 \\ \vdots & \vdots & \vdots & \vdots \\ 1 & \overline{p}^N & p_{wf}^N & \left(p_{wf}^k\right)^2 \end{bmatrix} \begin{bmatrix} a_0 & b_0 & c_0 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}$$

$$\text{...............................(A-15)}$$

Reservoir-Pressure Modeling. Recall a general material-balance equation for oil or gas reservoirs, in which the time-dependent pressure function is related to mass-cumulative quantities:

$$f\left[p(t)\right] = g\left(N_p, G_p, W_p, W_e\right), \quad \text{................................(A-16)}$$

$$\overline{p} = C_0 + C_1 \int q_o + C_2 \int q_w + C_3 \int q_g + C_4 \int q_{wi}, \quad \text{..............(A-17)}$$

$$\frac{d\overline{p}}{dt} = C'_1 q_o + C'_2 q_w + C'_3 q_g + C'_4 q_{wi}, \quad \text{........................(A-18)}$$

and the values of $C_0,\cdots,C_5,C'_1,\cdots,C'_4$ are the constants to fit units and history match. The equivalent discrete time and matrix form of the time-dependent pressure function is

$$\left[\left(\overline{p}\right)^k\right] = \begin{bmatrix} g_0 & g_1 & g_2 & g_3 & g_4 & g_5 \end{bmatrix} \begin{bmatrix} 1 & \overline{p}^{k-1} & q_o^{k-1} & q_w^{k-1} & q_g^{k-1} & q_{wi}^{k-1} \end{bmatrix}^T$$

$$\text{...............................(A-19)}$$

and the values of $g_0,\cdots,g_5$ are the resulting parameters after converting Eq. A-18 in the discrete-time form of Eq. A-19. Combining Eq. A-19 with fluid-rate models (Eqs. A-11 to A-13), it is also possible to get

$$\tfrac{1}{\Delta t}\left(\overline{p}^k - \overline{p}^{k-1}\right) \approx d_0 + d_1 \cdot \overline{p}^k + d_2 \cdot p_{wf1}^k + d_3 \cdot \left(p_{wf1}^k\right)^2 + d_4 \cdot p_{wf2}^k$$

$$+ d_5 \cdot \left(p_{wf2}^k\right)^2. \quad \text{.......................................(A-20)}$$

Therefore, for Δt=1, average reservoir pressure can be predicted as

$$\left(\overline{p}\right)^k = d_0 + d_1 \cdot \left(\overline{p}\right)^{k-1} + d_2 \cdot p_{wf1}^k + d_3 \cdot \left(p_{wf1}^k\right)^2 + d_4 \cdot p_{wf2}^k$$

$$+ d_5 \cdot \left(p_{wf2}^k\right)^2, \quad \text{...(A-21)}$$

and the values of $d_0,\cdots,d_5$ are the parameters to be fitted through regression. The equivalent matrix form of the time-dependent pressure function is

$$\left[\left(\overline{p}\right)^{k}\right]=\left[d_0 \quad d_1 \quad d_2 \quad d_3 \quad d_4 \quad d_5\right]\begin{bmatrix}1\\ \overline{p}^{k}\\ p_{wf}^{k}\\ \left(p_{wf1}^{k}\right)^2\\ p_{wf2}^{k}\\ \left(p_{wf2}^{k}\right)^2\end{bmatrix}. \quad \text{............(A-22)}$$

Appendix B—Multivariate MPC Formulation for Reservoir Management

Suppose that an oilfield process is described by some finite impulse-response model as

$$\mathbf{Y}^{k} = \mathbf{H}\mathbf{U}^{k:k-N}, \quad \text{..........(B-1)}$$

where

$$\mathbf{Y}^{k}=\left[y_1^k \quad y_2^k \quad \ldots \quad y_q^k \quad \ldots \quad y_{nY}^k\right]; q=1,2,\ldots,nY, \quad \text{.........(B-2)}$$

$$\mathbf{U}^{k}=\left[\mathbf{u}_1^k \quad \mathbf{u}_2^k \quad \ldots \quad \mathbf{u}_i^k \quad \ldots \quad \mathbf{u}_{nU}^k\right]; i=1,2,\ldots,nU, \quad \text{........(B-3)}$$

$$\mathbf{u}_i^{k}=\left[u_i^k \quad u_i^{k-1} \quad \cdots \quad u_i^{k-j} \quad \cdots \quad u_i^{k-N}\right]; j=1,2,\ldots,N, \quad \text{....(B-4)}$$

and $$y_q^k=\sum_{i=1}^{nU}\sum_{j=1}^{\infty}\left(h_{ij}^q\right)\left(u_i^{k-j}\right)\cong\sum_{i=1}^{nU}\sum_{j=1}^{N}\left(h_{ij}^q\right)\left(u_i^{k-j}\right). \quad \text{........(B-5)}$$

At time k, the following objective function must be minimized:

$$\min_{\Delta\mathbf{u}_i^{k+t-1}}\left\{\sum_{t=1}^{p}\left(\hat{\mathbf{Y}}^{k+t|k}-\mathbf{Y}^{SP}\right)^2+R\sum_{i=1}^{nU}\sum_{t=1}^{m}\left(\Delta\mathbf{u}_i^{k+t-1|k}\right)^2\right\}, \quad \text{..........(B-6)}$$

subject to the following constraints:

$$\mathbf{Y}_{\mathbf{min}}\le\hat{\mathbf{Y}}^{k+j|k}\le\mathbf{Y}_{\mathbf{max}};\ t=1,\cdots,p, \quad \text{..............................(B-7)}$$

$$\mathbf{u}_{\mathbf{min}}\le\mathbf{u}_{k+t-1|k}\le\mathbf{u}_{\mathbf{max}};\ t=1,\cdots,m, \quad \text{.............................(B-8)}$$

$$\mathbf{u}_{k+v|k}=\mathbf{u}_{k+m-1|k};\ v=m,\cdots,p-1. \quad \text{...............................(B-9)}$$

At time k, future process values can be estimated from the finite impulse-response model as

$$\hat{y}_q^{k+t|k}=\sum_{i=1}^{nU}\sum_{j=1}^{p}\left(h_{ij}^q\right)\left(u_i^{k-j+t}\right)+\hat{d}_q^{k|k}, \quad \text{............................(B-10)}$$

where

$$\hat{d}_q^{k|k}=y_q^k-\sum_{j=1}^{N}\left(h_{ij}^q\right)\left(u_i^{k-j+t}\right); t=1,\ldots,m,\ldots,p. \quad \text{...........(B-11)}$$

If we expand the future values of process output as

$$\mathbf{Y}_{\mathbf{F}}^{k}=\mathbf{H}\mathbf{U}^{k:k+p}, \quad \text{...(B-12)}$$

this could be rearranged in vector/matrix form as

$$\hat{\mathbf{Y}}_{\mathbf{F}}=\mathbf{H}_{\mathbf{F}}\mathbf{U}_{\mathbf{F}}+\mathbf{H}_{\mathbf{P}}\mathbf{U}_{\mathbf{P}}+\mathbf{Y}^{k}. \quad \text{....................................(B-13)}$$

The quadratic problem is set to find the minimum of

$$\begin{aligned}&\left(\hat{\mathbf{y}}_F-\mathbf{y}^{SP}\right)^T\left(\hat{\mathbf{y}}_F-\mathbf{y}^{SP}\right)+\Delta\mathbf{u}_F^T\mathbf{R}\Delta\mathbf{u}_F\\ &=\left(\mathbf{H}_F\mathbf{u}_F+\mathbf{H}_P\mathbf{u}_P+\mathbf{y}_k-\mathbf{y}^{SP}\right)^T\left(\mathbf{H}_F\mathbf{u}_F+\mathbf{H}_P\mathbf{u}_P+\mathbf{y}_k-\mathbf{y}^{SP}\right)\\ &+\left(\mathbf{D}\mathbf{u}_F+\mathbf{F}\mathbf{u}_P\right)^T\mathbf{R}\left(\mathbf{D}\mathbf{u}_F+\mathbf{F}\mathbf{u}_P\right)\\ &=\mathbf{u}_F^T\left(\mathbf{H}_F^T\mathbf{H}_F+\mathbf{D}^T\mathbf{R}\mathbf{D}\right)\mathbf{u}_F+2\mathbf{u}_F^T\left[\mathbf{H}_F^T\left(\mathbf{H}_P\mathbf{u}_P+\mathbf{y}_k-\mathbf{y}^{SP}\right)+\mathbf{D}^T\mathbf{R}\mathbf{F}\mathbf{u}_P\right]+\ldots\end{aligned}$$

$$\text{...............................(B-14)}$$

and linear constraints

$$\mathbf{y}_{\min}\le\mathbf{H}_F\mathbf{u}_F+\mathbf{H}_P\mathbf{u}_P+\mathbf{y}_k\le\mathbf{y}_{\max}, \quad \text{............................(B-15)}$$

$$\mathbf{u}_{\min}\le\mathbf{u}_F\le\mathbf{u}_{\max}, \quad \text{...(B-16)}$$

and $$\mathbf{U}\mathbf{u}_F=0, \quad \text{...(B-17)}$$

where

$$\mathbf{U}=\left[\begin{array}{c:cccc} & -1 & 1 & & \\ & & \ddots & \ddots & \\ {}_{(p-m)\times(m-1)} & & & -1 & 1_{(p-m)\times(p-m+1)}\end{array}\right]. \quad \text{.....(B-18)}$$

The above minimization is solved at each time k by finding the derivative of the quadratic problem as

$$\tfrac{\mathrm{d}}{\mathrm{d}\mathbf{u}}\left(\mathbf{u}^T\mathbf{A}\mathbf{u}+\mathbf{B}\mathbf{u}+\mathbf{C}\right)=2\mathbf{A}\mathbf{u}+\mathbf{B}=0\Rightarrow\mathbf{u}=\tfrac{1}{2}\mathbf{A}^{-1}\mathbf{B}, \quad \text{......(B-19)}$$

where

$$\mathbf{A} = \left(\mathbf{H}_F^T \mathbf{H}_F + \mathbf{D}^T \mathbf{R} \mathbf{D}\right) \quad \text{...(B-20)}$$

$$\mathbf{B} = 2\left[\mathbf{H}_F^T \left(\mathbf{H}_P \mathbf{u}_P + \mathbf{y}_k - \mathbf{y}^{SP}\right) + \mathbf{D}^T \mathbf{R} \mathbf{F} \mathbf{u}_P\right]. \quad \text{...............(B-21)}$$

The optimal value of u_k is sent to the process, and the process is left to run until time k+1, at which point the optimization problem is set up and solved again. The procedure is repeated indefinitely.

Nomenclature for Appendix B

$\mathbf{A}$ = Matrix quadratic term of $\mathbf{u}$ in the quadratic equation resulting of the grouping of B-14

$\mathbf{B}$ = Matrix linear term of $\mathbf{u}$ in the quadratic equation resulting of the grouping of B-14

$\mathbf{D}$ = Matrix of ones and zeros resulting of the expansion of B-13

$\hat{d}_q^{k|k}$ = Deviation between the actual process value of output signal q at time k and the estimated value of output signal q at time k.

$\mathbf{F}$ = Matrix of ones and zeros resulting of the expansion of B-13

$\mathbf{H}$ = Finite impulse-response model coefficients matrix

$\mathbf{H_F}$ = Expanded finite impulse-response model coefficients matrix required to build the future process values

$\mathbf{H_P}$ = Expanded finite impulse-response model coefficients matrix required to build the past process values

h_{ij}^q = Finite impulse-response model j^{th} coefficient of input signal i, for output signal q.

R = Suppression matrix, used to adjust proportionality gain to every input vector in the future moves.

$\Delta\mathbf{u}_i^{k+t-1|k}$ = Predictions of the variations (moves) to be made to input vectors ($\mathbf{u}_i^k$), at time $k+t-1$, made at time instant k.

$\mathbf{U}^{k:k-N}$ = Input signal matrix ($N \times nU$) containing input vectors ($\mathbf{u}_i^k$) at time k, and previous history up to k-N.

$\mathbf{U_F}$ = Input signal matrix (m-$1 \times nU$) containing the input vectors ($\mathbf{u}_i^k$) to be calculated by the MPC solution.

$\mathbf{U_P}$ = Input signal matrix ($N \times nU$) containing input vectors

$\mathbf{u}_P$ = Input signal matrix ($N \times nU$) containing the input vectors ($\mathbf{u}_i^k$) after decomposition.

$\mathbf{u}_F$ = Input signal matrix (m-$1 \times nU$) containing the input vectors ($\mathbf{u}_i^k$) to be calculated by the MPC solution after decomposition.

($\mathbf{u}_i^k$) at time k, and previous history up to k-N.

$\mathbf{u}_i^k$ = Input vectors ($\mathbf{u}_i^k$) of input signal i, at any time k and previous history up to k-N.

u_i^{k-j} = Scalar value of input signal i, at any time k-j

$\mathbf{u_{min}}$ = Constraints matrix (N×nU) containing minimum values of input signal i at anytime.

$\mathbf{u_{max}}$ = Constraints matrix (N×nU) containing maximum values of input signal i at anytime.

$\mathbf{Y}^k$ = Output signals vector (1×*nY*) at time instant *k*

$\hat{\mathbf{Y}}^{k+t|k}$ = Estimation of the future of output signals vector (1×*nY*) for time instant *k*+*t*, made at time instant *k*.

$\mathbf{Y}^{SP}$ = Vector of reference value (or setpoint) of output signals (1×*nY*)

$\mathbf{y}^{SP}$ = Vector of reference value (or setpoint) of output signals (1×*nY*) after decomposition

y_q^k = Scalar process value of output signal *q* at time *k*

$\hat{y}_q^{k+t|k}$ = Future process values of output signal *q* at time *k*+*t*, estimated at time *k*.

$\mathbf{Y}_{\mathbf{min}}$ = Constraints vector (1×*nY*) containing minimum values of output signal *q* at anytime.

$\mathbf{Y}_{\mathbf{max}}$ = Constraints vector (1×*nY*) containing maximum values of output signal *q* at anytime.

$\mathbf{Y}_{\mathbf{F}}^k$ = Future output signals vector (1×*nY*) at time instant *k*

$\hat{\mathbf{Y}}_{\mathbf{F}}$ = Vector of estimated future output signals (1×*nY*)

$\hat{\mathbf{y}}_F$ = Estimated future output signals vector (1×*nY*) after decomposition

Appendix C—Data Used for Examples

A data sample from a particular reservoir (within the El Furrial field) is used in this paper. Average reservoir data have been correlated previously with history-matched production; these data values have been selected or adjusted to ease research goals without misrepresenting field reality.

Rock and fluid data for examples in this paper resemble reservoir characteristics from El Furrial field, north of Monagas basin, Venezuela. El Furrial is a giant reservoir with 6 billion STB of 21°API original oil in place, currently producing approximately 500,000 STB/D from several independently hydraulic units.

Appendix D—Objective Function

The *objective function* (e.g., for a waterflooding project) may be expressed as the finite sum of discounted cash flows during the project horizon:

$$\text{NPV} = \sum_{k=1}^{N} \frac{\left[\left(q_o{}^k P_o + q_g{}^k P_g - q_{wp}{}^k C_{wp} - q_{wi}{}^k C_{wi}\right)\Delta T_k - I_T{}^k - C_F{}^k\right]\left(1 - r^k\right)}{\left(1+i\right)^{\frac{k \cdot \Delta T_k}{365}}}$$

……………………………(D-1)

where definitions are included in the Nomenclature. Net selling revenues of oil and gas take into account the selling price minus the associated production costs such as operational budget (payroll, supplies, maintenance, treatment, and transport) and production royalties.

In practice, the way we achieve the optimal solution to Eq. 1 is that we assume a time model for $q_o{}^k$, $q_g{}^k$, $q_{wi}{}^k$, and $q_{wp}{}^k$ that allows us to find the cash flows in time for certain assumptions in C_{wp}, C_{wi}, $I_T{}^k$, $C_F{}^k$, and i and, ultimately, find a maximum (or minimum) value of Eq. D-1 while honoring system constraints.

In the case of reservoir exploitation, these constraints are imposed by the reservoir, wells, surface equipment, cost, and schedule. For example, in finding the solution for optimizing the production rate of a well, the physical constraints are given by the reservoir productivity (IPR curves) and the tubing performance, as shown in Fig. 6. For example, this figure shows the operating region of an oil well defined by two IPR curves and two tubing-performance curves. The IPR curves (IPR_1, IPR_2) represent reservoir conditions at different static pressure (P_{RES1}, P_{RES2}) and well productivity indexes. Tubing performance is defined for two distinctive operating conditions of GOR (GOR_1, GOR_2), water fraction (f_{w1}, f_{w2}) and tubinghead pressure (p_{THP1}, p_{THP2}). Points 1, 2, 3, and 4 in Fig. 6 define the region of operability for this particular example.

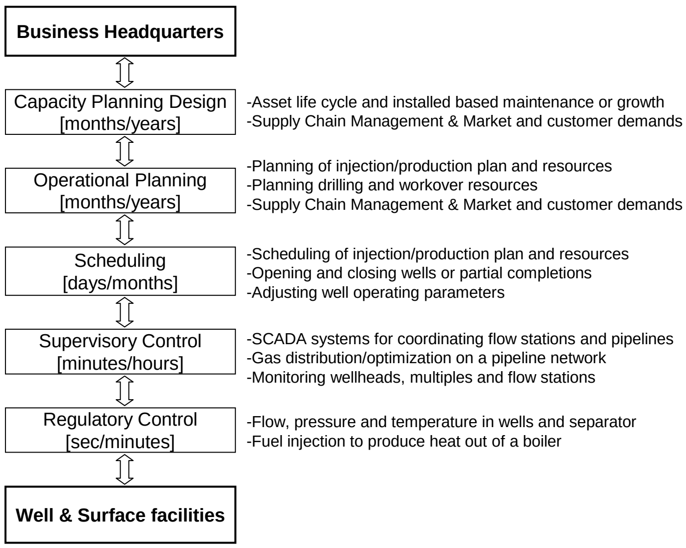
$$\begin{cases} p_{wf}, \max \geq p_{wf} \geq p_{wf}, \min \\ q_l, \max \geq q_l \geq q_l, \min \end{cases} \quad . \quad \text{................................(D-2)}$$

The shaded area in Fig. 16 denotes the region of feasible solutions.

Depending on the models used in Eq. D-1, the objective function may be a linear or nonlinear function. Several methods may be used for different situations.

SI Metric Conversion Factors

°API	141.5/(131.5+°API)	= g/cm^3
bbl	$\times$ 5.165	= ft^3
cp	$\times$ 10^{-3}	= Pa·s
ft	$\times$ 3.048 x 10^{-1}	= m
ft^2	$\times$ 9.290304 $\times$ 10^{-2}	= m^2
ft^3	$\times$ 2.831685 $\times$ 10^{-2}	= m^3
°F	(°F – 32)/1.8	= °C
in.	$\times$ 2.54	= cm
psi	$\times$ 6.894757	= kPa
sq mile	$\times$ 2.589 988	E + 00= km^2



Fig, 1—Field operations hierarchy.[18]

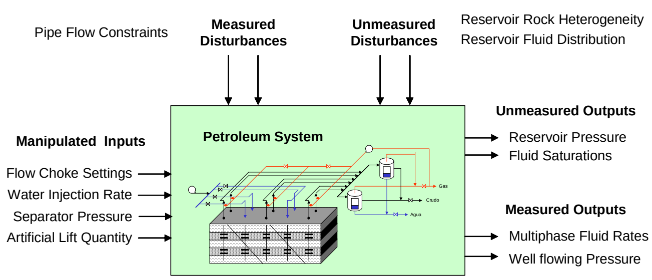


Fig. 2—Petroleum system represented as a multiple-inputs and multiple-outputs (MIMO) structure.

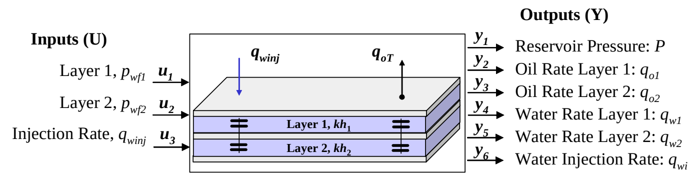


Fig. 3—Waterflooded reservoir for MPC example.

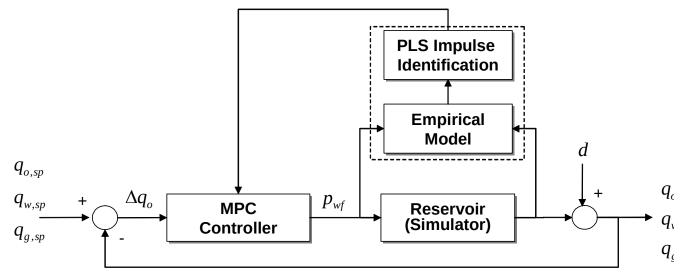


Fig. 4—MPC level and simultaneous identification implementation diagram.

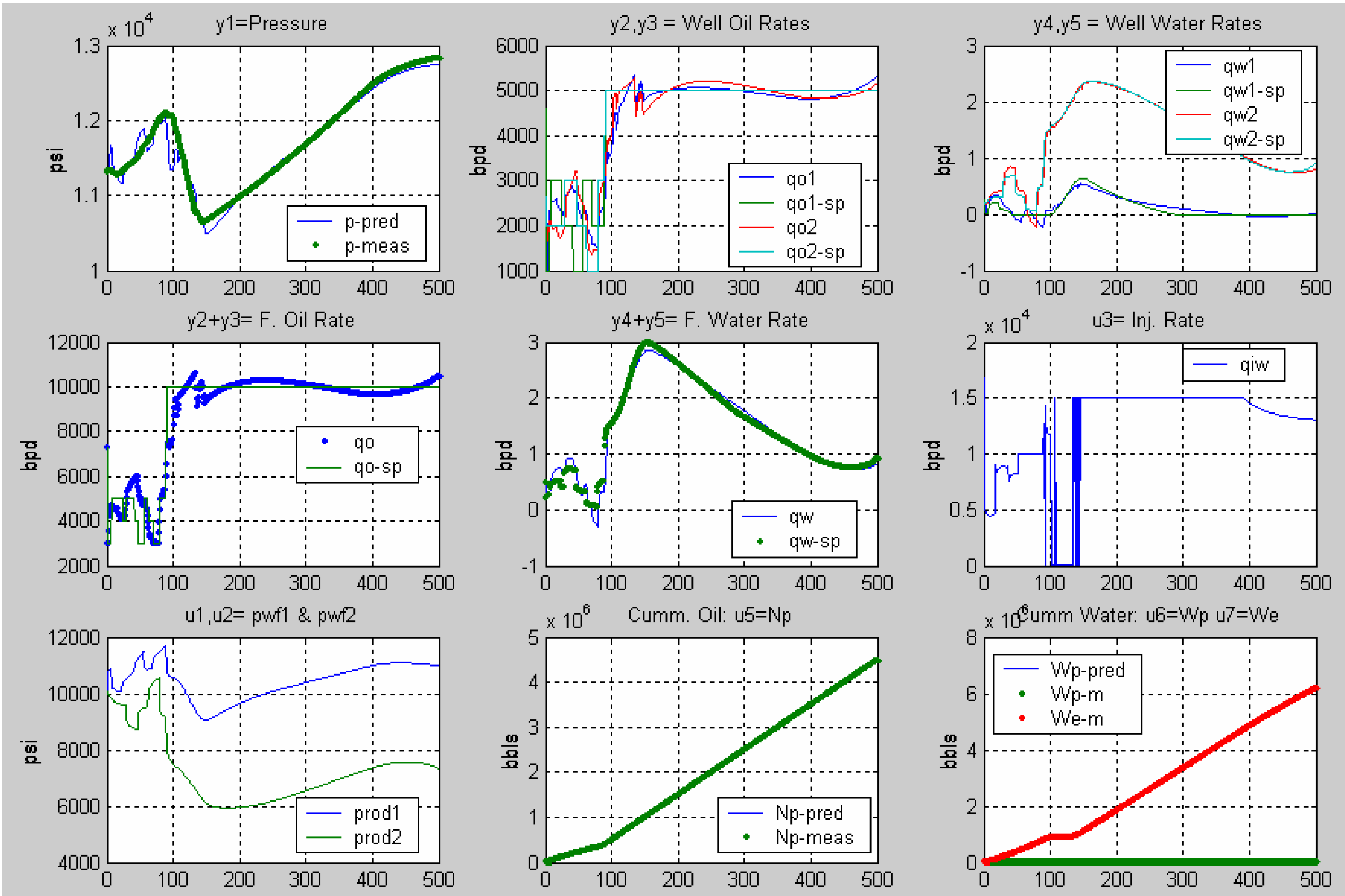


Fig. 5—MPC injector/producer problem.

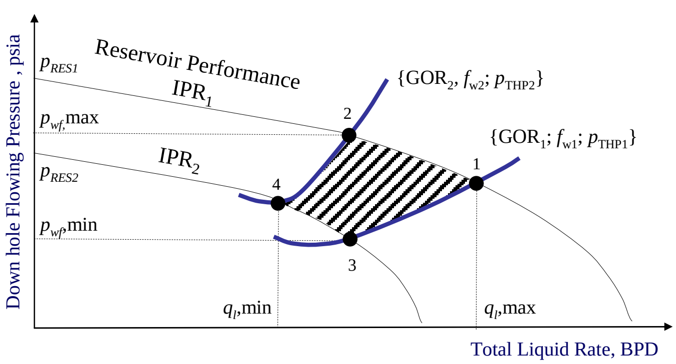


Fig. 6—Well constraints vs. vertical-lift performance.

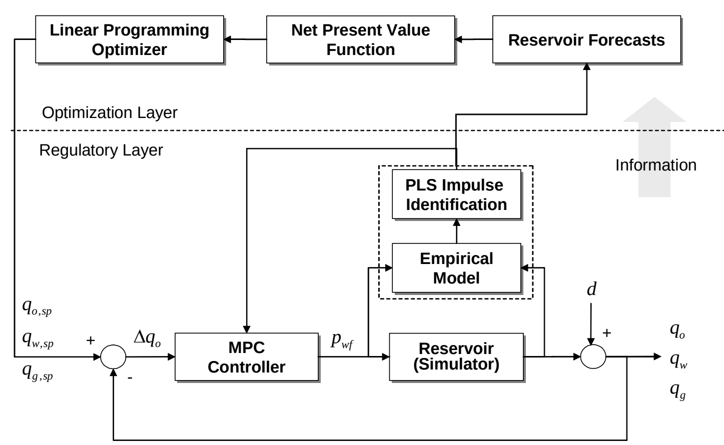


Fig. 7—Bilevel self-learning reservoir control diagram.

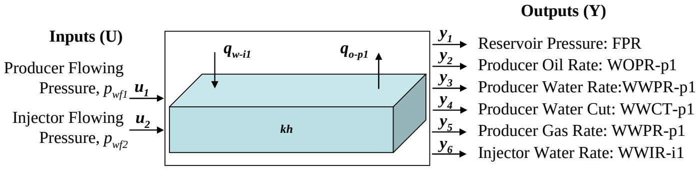


Fig. 8—Waterflooded reservoir model for performance-prediction example. FPR = field pressure (psia), WOPR = Well oil-production rate (STB/D), WWPR = Well Water-Production Rate, STB/D , WWCT = Well Water Cut, fraction, and WWIR = Well Water Injection Rate, STB/D
.

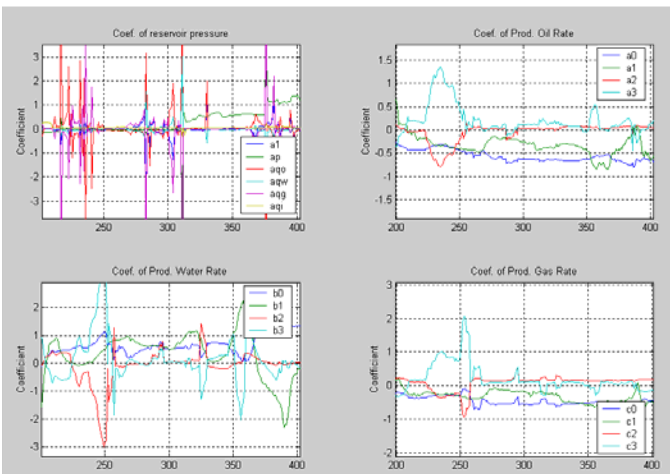


Fig. 10—Learning parameter evolution with time.

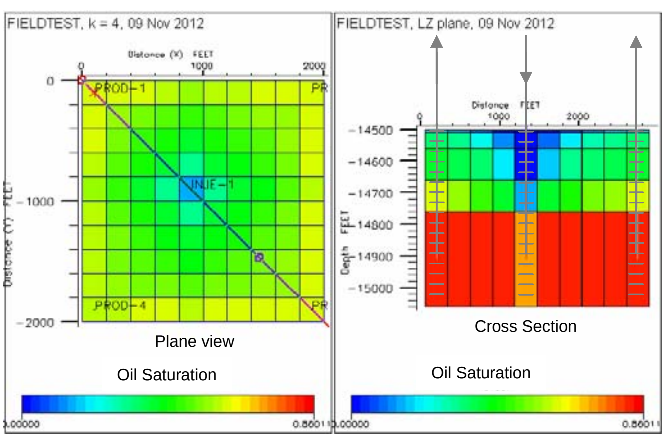


Fig. 11—Five-spot waterdrive problem under no control management.

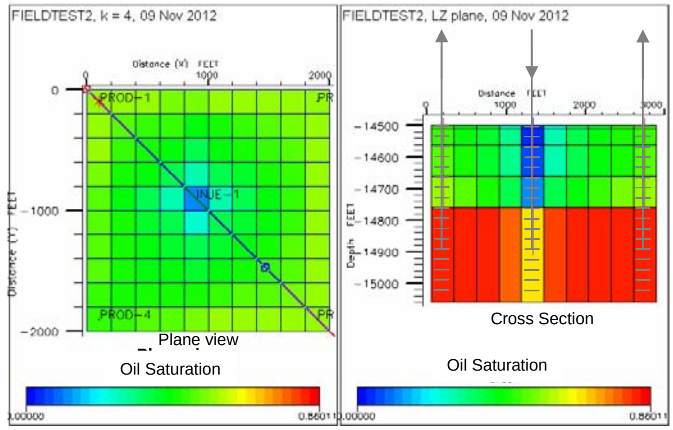


Fig. 12—Self-learning reservoir management applied to five-spot waterdrive problem.

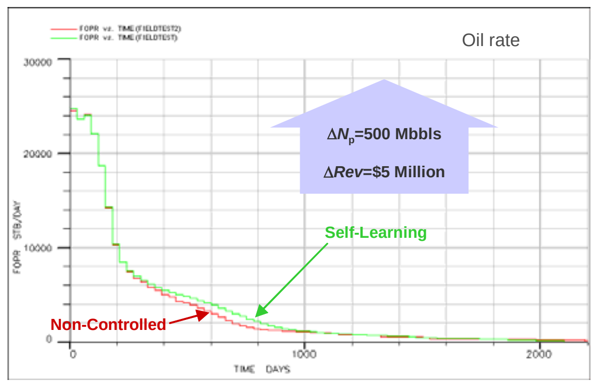


Fig. 13—Oil-rate comparison (five-spot waterdrive problem). FOPR = field oil-production rate (STB).

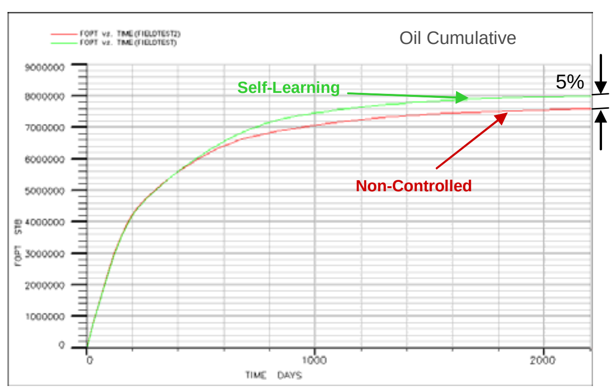


Fig. 14—Oil cumulative comparison (five-spot waterdrive problem). FOPT = field oil-production cumulative (STB).

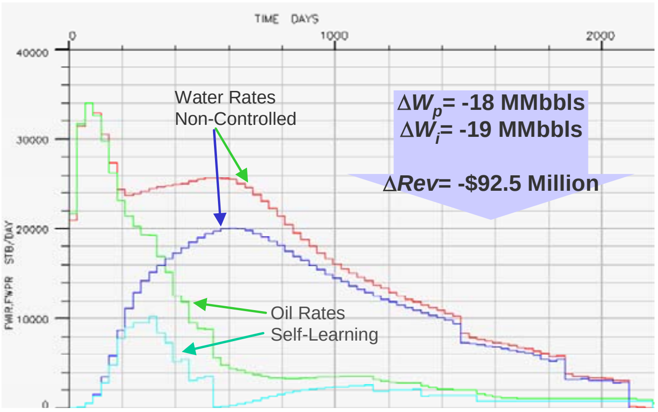


Fig. 15—Water-rate comparison (five-spot waterdrive problem). FWIR = field water-injection rate (STB/D), FWPR = field water-production rate (STB/D).

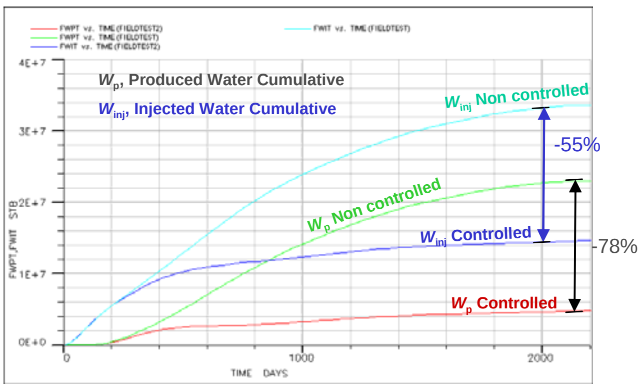


Fig. 16—Water cumulative (five-spot waterdrive problem). FWPT = field water-production rate (STB/D), FWIT = field water-injection cumulative.